CONVOLUTIONAL CODES FOR CPM USING THE MEMORY OF THE MODULATION PROCESS

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ABSTRACT

The modulation process of continuous phase modulation (CPM) is interpreted as a redundant encoder with memory and a signal table. It is shown that by a fusion of this encoder with a preceding convolutional encoder, the total number of delay elements and by this the number of states, which have to be traced at the receiver, can be reduced. At first a mapping rule is used between the codewords and the signal, for which the memory of the modulation process can be expressed by a linear, nonrecursive circuit, and therefore leads to an obvious form of the fusion of both encoders. Thereafter the method is extended to mapping rules with a recursive memory structure and arbitrary phase pulses.

I. INTRODUCTION

In several papers (e.g.|2|-|5|) convolutional codes are given, with which the reliability of digital modulation with continuous phase is increased. The spectral efficiency of these schemes is conserved by transmitting the codes' redundancy by means of an increased signal space. In this paper it is shown that the inherent memory of the modulation process can be used by the convolutional encoder. This results in a decreased number of encoder-modulator states, which have to be traced at the receiver or an increased coding gain respectively. For that purpose the modulation process is divided into a redundant encoder with memory and a signal table (cf.|6|). This point of view leads to many others than the usual mapping rules between the codewords and the signal. On the other hand the knowledge of the internal structure of the memory of the modulation process makes plain redundant encoder- modulator states in a fusion of both encoders. At first the method is explained only for CPFSK-signals (|1|). In section VI the results are generalized to CPM with other baseband phase pulses. For several examples optimum codes and coding gains are given.

II. SIGNAL DESCRIPTION AND DEFINITIONS

The same block diagramm (Fig. 1) and signal description of coded CPM are used as in ref. |2|, |4|. Each time interval a convolutional encoder |10| with ν_c binary delay elements and rate $r_c = k/n$ generates a codeword $\vec{x} = (x_o,..,x_{n-1})$ with binary symboles x_i from k binary source symboles $\vec{u} = (u_o,..,u_{k-1})$. In a mapper, which in contrast to |2|-|5| may contain memory itself, M-ary phase transition factors $\alpha_i \in \{\pm1,\pm3,...,\pm(M-1)\}$ are coordinated to the codewords $\vec{x}$. The CPM-modulator produces the transmitter signal:

$$s(t,\langle\alpha\rangle) = \sqrt{\frac{2E}{T}}\cos(\varphi(t,\langle\alpha\rangle)) \qquad (1)$$

E: Signal energy per symbol interval T

The phase function $\varphi(t,\langle\alpha\rangle)$ is given by:

$$\varphi(t,\langle\alpha\rangle) = 2\pi f_c t + \varphi_o + 2\pi h \sum_{i=o}^{\infty} \alpha_i \cdot q(t-iT) \qquad (2)$$

f_c: Carrier frequency

h: Modulation index

q(t): Phase pulse with the restriction:

$$q(t) = \begin{cases} 0, & t < 0 \\ 1/2, & t \geq LT \end{cases}$$

A slope-function $c_L(t)$ is used:

$$c_L(t) := \begin{cases} 0 & t < 0 \\ \frac{t}{2LT} & 0 \leq t < LT \\ 1/2 & t \geq LT \end{cases} \qquad (3)$$

For sections III to V a phase pulse $q(t) = c_1(t)$ is presumed (CPFSK-modulation).

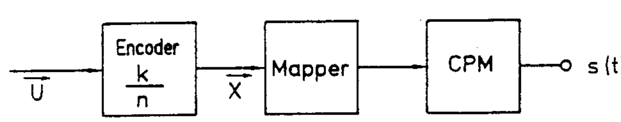

Fig. 1 Block diagram

For simplification following notation is used for the phase function eq. 2 within the time interval $mT \leq t < (m+1)T$, $m \geq L$ (cf.|7|):

$$\varphi(t,\langle\alpha\rangle) = 2\pi f_c t + \varphi_o - 2\pi h \sum_{i=o}^{m}(M-1)\, c_L(t-iT) +$$

$$+ 2\pi h \sum_{i=o}^{m}(\alpha_i q(t-iT) + (M-1)\, c_L(t-iT))$$

$$= 2\pi f_R t + \varphi_{oR} + \Theta_{m-L} + 2\pi h \sum_{i=m-L+1}^{m} b(\alpha_i, t-iT) \qquad (4)$$

with:

reference frequency

$$f_R = f_c - \frac{h(M-1)}{2\,T}\,, \qquad (5)$$

phase-state

$$\Theta_{m-L}=(\pi h \sum_{i=o}^{m-L} (\alpha_i+M-1)) \bmod 2\pi, \qquad (6)$$

and phase transition function

$$b(\alpha,t) = \begin{cases} \alpha q(t)+(M-1)c_L(t) & t < LT \\ o & t \geq LT. \end{cases} \qquad (7)$$

All frequency or phase deviations Δf, $\Delta\varphi$ are to be interpreted relative to the reference frequency f_R or reference phase

$$\varphi_R(t) = 2\pi f_R t + \varphi_{oR}.$$

For rational modulation indicies $h = p/q$ there are q different phase states Θ_{m-L} relative to the reference phase (p,q integers with no common divisor).

The optimization of the convolutional codes and the mapping rule is accomplished by aid of the minimum normalized squared Euclidean distance (cf.|2|) between two signals s(t) for different source symbol series. The performance of the coding-modulation scheme disturbed by white Gaussian noise is described by this parameter with sufficient exactitude (|8|).

III. EXTRACTION OF THE MEMORY OF THE MODULATION PROCESS AND MAPPING RULES

For phase pulses $q(t) = c_1(t)$ there are M•q different phase trajectories within one time interval. Subsequently these phase trajectories will be called signal elements of the signal set. They are addressed by M different instantaneous frequency deviations Δf (phase transition factors α) and q phase states Θ_{m-1} (phase at the beginning of the symbol interval). (There are other methods for addressing the signal elements too: (Θ_{m-1}, Θ_m), $(\Delta f, \Theta_m)$.)

Fig. 2 shows a graphic representation of the signal elements for the example M = 4, h = 1/4.

The modulation process can be divided into a redundant encoder E_M with memory and rate $r_M = ld(M)/ld(M\cdot q)$, which guarantees the phase continuity and a table of the M•q signal elements. Within this encoder $\nu_M = [ld(q)]$ binary delay elements contain in the influence of past signal elements on the phase state ([x]: next integer ≥ x).

As an example Fig. 3 shows such a desintegration of a CPFSK-modulator for M = 4, h = 1/4. The instantaneous frequency deviation is addressed by the binary symbols y_o, y_1, the phase state by y_2, y_3. As the phase state is a function of all past symbols α_i (eq.6), the encoder E_M has a recursive structure.

For CPFSK all signal elements with the same frequency deviation - the elements of one row in Fig. 2 - are mapped to one codeword $\vec{x}$, which will be called "frequency-mapping". But there are many other possibilities. For a specific mapping rule the set of M•q signal elements must be partitioned into M classes,

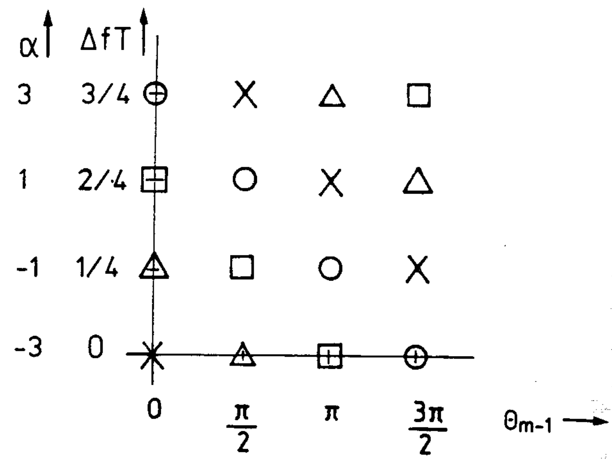

Fig. 2 Signal elements for L=1, M=4, h=1/4

each with q elements. The classes are mapped to the different modulator inputs $\vec{x}$. To guarantee phase continuity each class has to cover all possible phase-states. Thus, the elements of a class have to be selected all from different columns in Fig. 2. Therefore $(M!)^{q-1}$ different possibilities exist for partitioning the signal elements into M classes. As M! mapping rules exist between the codewords and the classes for one specific partitioning (cf.|2|), there are $(M!)^q$ mapping rules in total.

If the restriction to frequency-mapping is omitted, the optimization problem for the encoder and the mapping rule gets enormous additional degrees of freedom. Although many of these mapping rules have identical properties (|2|), it is quite impossible to take all cases into account.

IV. PHASE-STATE-MAPPING AND CONVOLUTIONAL CODES

In this section a mapping rule is considered for M = q, in which the information is expressed by the phase at the end of a symbol interval, which is equivalent to the phase-state Θ_m for the next symbol $\vec{x}_{m+1}$ i.e.(Θ_{m-1} for input symbol $\vec{x}_m$). The phase continuity is performed by the frequency deviation. Thus, phase and frequency change their roles as slave and master compared to frequency-mapping. (For the example of Fig. 2 this "phase-state-mapping" corresponds to a partitioning of the signal elements into classes, which is characterized by identical signs for the signal elements.)

For L=1 the selection of a signal element is only a function of the actual and the preceding modulator input word $\vec{x}$. Thus, for phase-state-mapping the encoder E_M has a structure as indicated in Fig. 4. The part with memory is nonrecursive and linear in GF(2), wheareas the memoryless (normally nonlinear) combinatorial circuit depends on the special mapping rule and the method of addressing the signal elements.

If some output lines y_o to $y_{(ldM-1)}$ of the encoder E_M address the instantaneous frequency deviation of the signal elements (the phase transition factors α respectively), a CPFSK-mo-

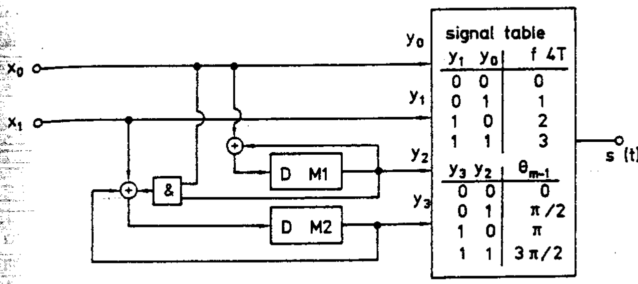

Fig. 3 Desintegration of a CPFSK-modulator into an encoder E_M with memory and a signal table (M=4, h=1/4)

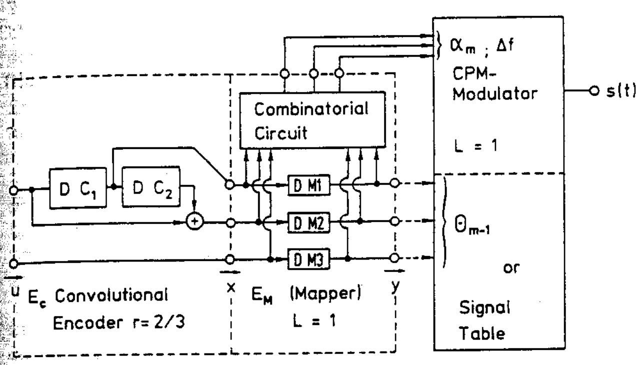

Fig. 4 Convolutional encoder E_C and "mapper" E_M for phase-state-mapping

dulator may be used for the signal generation instead of the signal table. As in this case the phase continuity is guaranteed by the modulator itself, the remaining output lines of E_M are irrelevant. Thus, the encoder E_M of Fig. 4 can be interpreted as a mapper with memory, which converts frequency-mapping to phase-state-mapping. The memory of this mapper contains the identical phase-state information as the internal memory of the CPFSK-modulator. Therefore the number of states of the modulation process is not increased by this mapping with memory.

Together with the preceding convolutional encoder E_C the total number of different states is less than the product of the state numbers of this encoder and of the modulation process, if the combination of both encoders E_C and E_M contains redundant delay elements. Such redundant delay elements occur, if the inputs of the memoryless part of the modulator (signal table and combinational circuit) are not generated from the source symbols by a "canonical" circuit with minimal number of delay elements. This fact may be interpreted as a usage of the inherent memory of the modulation process by the convolutional code. (In the example of Fig. 4 the delay elements C2 and M1 both contain the past information symbol u_o.)

On the other hand the combination of E_C and E_M can be seen as a nonlinear trellis encoder (because of the nonlinear combinatorial circuit) for CPFSK with ν_M binary delay elements, which are synchronous to the inherent memory of the modulation process.

For phase-state-mapping the selection of a signal element is a function of the n binary modulator input series and their delayed versions. In the nonrecursive,obvious and minimal encoder realization the memory of the convolutional code (cf. |10|) is represented by k shift registers, one for each encoder input, with total ν_c binary delay elements. If each of the shift registers is extended by one additional delay element, all ld(M•q) input signals of the memoryless combinatorial circuit and the signal table (Fig. 4) can be generated from these k shift registers using the generator polynomials of the convolutional encoder and versions multipled by D. Thus, ν_c + k binary delay elements are sufficient for coding and modulation in such a canonical structure. Therefore n-k binary delay elements are redundant for phase-state-mapping.

V. FREQUENCY-MAPPING AND CONVOLUTIONAL CODES

There is a disadvantage of phase-state-mapping compared to frequency-mapping, because for frequency-mapped signals only the differences between phase states have to be traced at the receiver, but no specific relation to the reference phase is necessary. So frequency-mapping rules have a natural property, which is comparable to the rotationally invariance of some coded PSK systems |9|. (From Fig. 2 it is recognizable that only for frequency-mapping rules the coordination of a signal element to a class is independent of the phase-state. Perhaps there are special codes, with which rotationally invariant schemes arise for other mapping rules analogous to |9|). For the decomposition of the modulation process for frequency-mapping the structure of the encoder E_M can be derived directly from eq. 6 (L=1). For this use the different phase-states θ_{m-1} are represented by integer numbers 0,1,...(q-1). The next phase-state is the sum of the past phase-state θ_{m-2} and the nonnegative integer $p\cdot(\alpha_{m-1}+M-1)/2$ with a subsequent reduction modulo q. Representing the phase-state by ν_M binary digits, the recursive structure given in Fig. 5 arises for the encoder E_M.

An obvious identification of redundant delay element within the CPM-scheme combined with a convolutional codes is possible, if a part of this encoder E_M is expressable by a switching circuit linear in GF(2), which is characterized by $g_M(D) = P(D)/Q(D)$. (In the example Fig. 3 the output series $\langle y\rangle_2$ is given by $y_2(D) = x_o(D)\cdot D/(1+D)$.) If the generator polynomials of the minimal and obvious realization of the convolutional encoder E_c, which contributes to the inputs of the modulator with this linear phase-state memory, contain Q(D) (or factors of it) as factors, the corresponding phase-state address is generated by a noncanonical circuit and therefore redundant delay elements exist.

For the "natural" frequency-mapping rule (Q1 in |2|: increasing α mapped to increasing inputs $\vec{x}$, interpreted as dual numbers), a linear description for one delay element of the phase-state memory is possible for modulation indices h = p/q with q even (p odd). For the LSB of θ_{m-1} no carry handling and for q even no reset operation due to the reduction modulo q is necessary. The linear structure D/(1+D) for the input x_o of the modulator is shown in Fig. 6. Thus, all convolutional codes, for

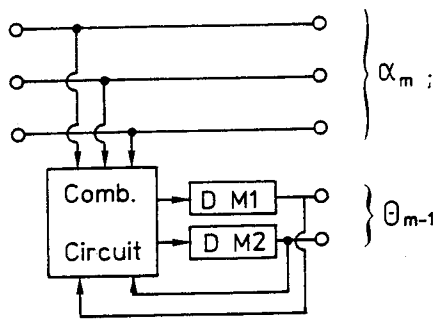

Fig. 5 Structure of E_M for frequency-mapping

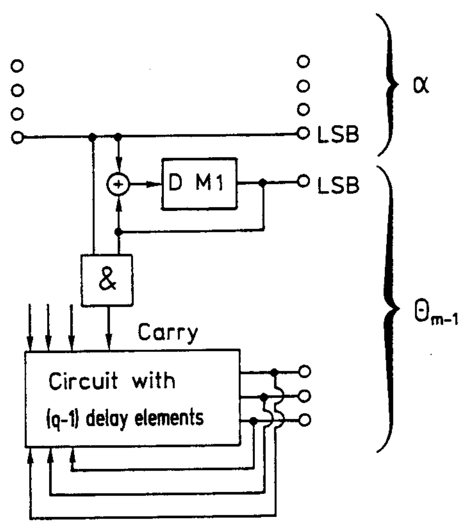

Fig. 6 Encoder E_M for frequency-mapping with a linear part for the LSB of the phase-state address (q even)

which the polynomials generating the output x_o have even numbers of coefficients one, use one binary delay element in common with the modulation process.

VI. EXTENSION TO CPM WITH ARBITRARY PHASE PULSES

For L = 1, but $q(t) \neq c_1(t)$ the only difference to the previous sections is that a phase transition function $b(\alpha,t)$ (eq.7) instead of a instantanous frequency deviation is addressed by the phase transition factors α or the address lines y_o to y_{ldM-1} (cf. Fig. 3). By this means the representation of signal elements in Fig. 2 has a valid interpretation for arbitrary phase pulses too. The encoder E_M is not affected by the special shape of the phase pulse.

For partial response signals (L>1) there are up to $q \cdot M^L$ different signal elements within one time interval T. This signal elements can be addressed by the phase-state Θ_{m-L} and L phase transition factors α_{m-L+1} to α_m corresponding to the superposition of phase transition functions (eq.4).

For frequency-mapping the decomposition of the modulation process arises from eq.4 and is given in Fig. 7. The first nonrecursive part of the encoder E_M handles the superposition of phase pulses; the second recursive

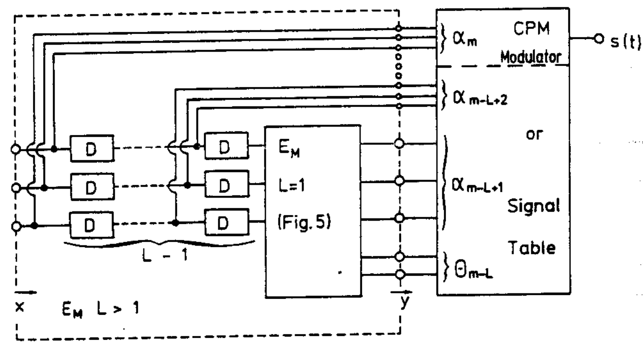

Fig. 7 Desintegration of the CPM-modulator for phase pulses with L>1 (frequency-mapping)

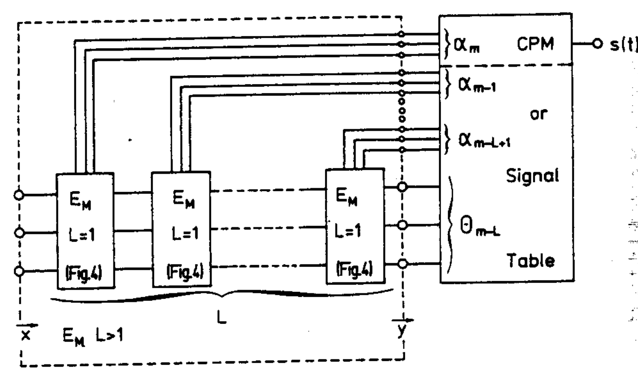

Fig. 8 Desintegration of the CPM-modulator for phase pulses with L>1 (phase-state-mapping)

part belongs to the sequence of phase-states and is independent of L (cf. eq.6). The outputs of the n(L-1) delay elements of the nonrecursive part can be generated from k(L-1) delay elements, added to the k shift registers of a preceding convolutional encoder with minimal and obvious realization (cf. section IV). Thus, $(n-k)\cdot(L-1)$ delay elements of the combined encoder-modulator scheme are redundant within the nonrecursive part (cf.|5|). For q even, natural mapping, and generator polynomials with a even number of nonzero coefficients for the encoder output x_o one additional common delay element exists in the same way as for L=1.

For phase-state-mapping the encoder E_M consists of L encoders for L=1, each of them generating one phase transition factor α of the address of the signal elements (Fig. 8). Thus $(n-k)\cdot L$ delay elements of the combined encoder-modulator scheme are redundant. If a CPM-modulator is used instead of a signal table for the signal generation, the "mapper" E_M with memory (Fig. 4), which converts frequency-mapping to phase-state-mapping for L=1, performes this conversion for arbitrary phase pulses too (cf. Fig. 8).

VII. RESULTS

In table 1 to 3 some optimum convolutional codes and minimum squared Euclidean distances are listed for CPM with phase-state-mapping and frequency-mapping in octal numbers (x_1, x_0) (cf. |2|). In the columns "phase-state-mapping" and "frequency-mapping with fusion" codes are given, which use L delay elements in common with the modulation process.

The columns "frequency-mapping without fusion" contain codes with L-1 common delay elements. Therefore these codes, which are given for comparison and are partially taken from ref. |2|, |3|,|5|,have one delay element less than the other codes. But the total number 2^{ν} of encoder-modulator states is identical for each row of the tables. If no improvement is possible by a usage of the memory of the modulation process no code is listed.

For $q(t)=c_1(t)$ (CPFSK), $k/n=1/2$, $h=1/4$, there are remarkable improvements of phase-state-mapping to frequency-mapping for $\nu = 3$, 5, and 7. (A free optimization of labeling the 16 signal elements to the pathes of a trellis with 2^{ν} states has given that for $\nu = 3$ and $\nu = 4$ there are no better, but many equivalent schemes as long as phase continuity is presumed).

The usage of a common delay element for frequency-mapping gives gains for $\nu = 4$, $\nu = 7$ and for $h = 1/2$ for $\nu = 2$.

For the codes of rate 2/3 and CPFSK $M = 8$, $h = 1/8$ the additional source symbol u_1 is mapped without coding to x_2(cf. Fig. 4). In this case the total improvement of one additional delay element is possible by phase-state-mapping.

Tables 2 und 3 contain results for partial response phase pulses of the type "raised-cosine" (L-RC cf. |1|).

Approximately the same improvements are obtained by phase-state-mapping as for CPFSK (1REC). In one case almost the same minimum Euclidean distance is achieved with frequency-mapping. Therefore it is not clear, what is the prize for rotational invariance. (For 3 RC some reference values differs to those given in |5|.)

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Table 1: Some optimum codes and minimal Euclidean distances, frequency impuls: 1REC ($q(t)=c_1(t)$)

ν	M=4, h=1/2, r_c=1/2 frequency-mapping (with fusion) code (x_1,x_0)	d^2_{min}	(without fusion) code (x_1,x_0)	d^2_{min}	M=4, h=1/4, r_c=1/2 phase-state-mapping code (x_1,x_0)	d^2_{min}	frequency-mapping (with fusion) code (x_1,x_0)	d^2_{min}	(without fusion) code (x_1,x_0)	d^2_{min}	M=8, h=1/8, r_c=2/3 phase-state-mapping code (x_1,x_0)	d^2_{min}	frequency-mapping (with fusion) code (x_1,x_0)	d^2_{min}	(without fusion) code (x_1,x_0)	d^2_{min}
2	(1,5)	4	(1,2)*	3*												
3	--	--	(1,7)*	5*	(5,2)	3,58	--	--	(1,2)*	3,00*						
4	--	--	(1,15)*	6*	(13,4)	4,51	(13,6)	4,42	(7,2)*	4,30*	(5,2)	2,55	--	--	(2,1)*	1,47*
5	--	--	(4,23)*	7*	(23,4)	5,81	--	--	(13,2)*	5,24*	(15,2)	3,13	--	--	(5,2)*	2,55*
6	--	--	(44,13)	8	(43,10)	6,18	--	--	(23,10)*	6,15*						
7					(107,10)	7,09	(127,14)	6,73	(57,16)	6,45						

*from reference /2/,/3/.

Table 2: Frequency impuls $\dot{q}(t)$: 2RC

ν	M=4, h=1/2, r_c=1/2 frequency-mapping (with fusion) code (x_1,x_0)	d^2_{min}	(without fusion) code (x_1,x_0)	d^2_{min}	M=4, h=1/4, r_c=1/2 phase-state-mapping code (x_1,x_0)	d^2_{min}	frequency-mapping (with fusion) code (x_1,x_0)	d^2_{min}	(without fusion) code (x_1,x_0)	d^2_{min}
3	(4,5)	3,74	(2,1)*	3,04*						
4	(2,11)	4,79	(4,7)*	4,75*	(5,2)	3,25	--	--	(2,1)*	2,98*
5	(4,21)	5,93	(3,13)$^{\Delta}$	5,88*	(15,2)	4,14	(13,6)	4,12	(7,2)*	3,71*
6	--	--	(34,15)	6,94	(23,4)	5,45	(31,6)	4,93	(13,4)*	4,80*
7	--	--	(74,31)	8,01	(51,4)	5,83	--	--	(31,2)	5,51

*from reference /5/. $^{\Delta}$different to reference /5/.

Table 3: Frequency impuls $\dot{q}(t)$: 3RC

ν	M=4, h=1/2, r_c=1/2 frequency-mapping (with fusion) code (x_1,x_0)	d^2_{min}	(without fusion) code (x_1,x_0)	d^2_{min}	M=4, h=1/4, r_c=1/2 phase-state-mapping code (x_1,x_0)	d^2_{min}	frequency-mapping (with fusion) code (x_1,x_0)	d^2_{min}	(without fusion) code (x_1,x_0)	d^2_{min}
4	--	--	(2,1)	3,39$^{\Delta}$						
5	--	--	(4,1)*	4,51*	--	--	--	--	(2,1)	2,86$^{\Delta}$
6	--	--	(10,13)$^{\Delta}$	5,29$^{\Delta}$	(13,4)	3,51	(13,3)	3,07	(5,2)	2,43
7	--	--	(33,13)	6,15	(31,4)	4,27	--	--	(13,2)$^{\Delta}$	3,58*

*from reference /5/. $^{\Delta}$different to reference /5/.