Iterative Equalization with Soft Feedback with a Subsequent Stage Utilizing Hopfield Networks for Error Search and Correction

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Abstract

In this paper, a second stage for algorithms applying iterative soft decision interference cancellation (ISDIC) [1], [2], [3], [4] for channel equalization is proposed performing error search and correction. Analysis by simulations shows that a matched filter (MF) ISDIC with subsequent second stage can outperform a more complex minimum mean–squared error (MMSE) ISDIC. Furthermore, the latter can be improved by the proposed second stage by up to 2 dB. It is shown that the ISDIC scheme with following second stage can reach the matched filter bound up to a fraction of a dB for all analyzed channels and 4QAM transmission and up to 1 dB for 16QAM transmission. In [1] it has been reported that the MF ISDIC performs very well for highly dispersive channels, e.g. with 60 symbol–spaced paths, but for channels of moderate dispersion an error floor occurs. Utilizing performance bounds for delayed decision–feedback sequence estimation (DDFSE) we show that now already for channels of moderate length, e.g. 20 symbol–spaced taps, even the low–complexity MF ISDIC with the proposed subsequent second stage outperforms a DDFSE approach with about 10^9 states.

1 Introduction

It is well known that maximum–likelihood sequence estimation (MLSE) using the Viterbi algorithm (VA) [5] is an optimum scheme for equalization of dispersive channels producing intersymbol interference (ISI). However, the complexity of the VA becomes prohibitively high for long channel impulse responses (CIRs), and suboptimum schemes have to be applied in this case, like decision–feedback equalization (DFE), delayed decision–feedback sequence estimation (DDFSE) [6], or reduced–state sequence estimation (RSSE) [7]. Because a minimum–phase impulse response is essential for suboptimum trellis–based equalizers, in general, a prefilter is necessary which transforms the CIR into its minimum–phase equivalent [8]. Even with optimized prefilter, a very high number of states might be necessary for reduced–state equalizers such as DDFSE or RSSE in order to approach the performance of MLSE.

In [1], an iterative soft decision interference cancellation (ISDIC) algorithm for binary phase–shift keying (BPSK) transmission has been introduced, which requires no minimum–phase response but employs a matched filter (MF) as a front end and performs even better than optimized DDFSE with high number of states. However, this holds only for long random CIRs with e.g. 60 symbol–spaced taps. For CIRs of moderate length and/or with well–defined shape, an error floor occurs and the performance of MLSE cannot be achieved.

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In [2], the algorithm of [1] has been generalized to 4QAM (quadrature amplitude modulation) transmission and then extended by using minimum mean–squared error (MMSE) filtering instead of a MF. With this modification, high performance is also obtained for CIRs of only moderate length. A similar MMSE–based scheme has been proposed in [9] for synchronous CDMA transmission. But contrary to the MMSE scheme proposed in [2], ISI was not considered and the width of the MMSE filter was equal to the spreading gain whereas in [2] a sliding window MMSE filter has been used resulting in a noticeably lower complexity compared to filtering of a whole transmission block, while high performance is maintained.

In this paper, we give insight into the error patterns occurring when using an ISDIC algorithm and utilize this knowledge to derive a second stage for ISDIC algorithms performing error search and correction. This second stage is based on Hopfield networks [10], [11], [12] which converge to a local minimum of an optimization problem [11], [12]. The considered transmission system is shown in Fig. 1 and is described in detail in the subsequent sections.

The paper is organized as follows. First, we introduce the system model in Section 2. In Section 3 we review the MF ISDIC according to [1], [2], [13] and the MMSE ISDIC according to [2], [3], [4]. The proposed second stage for the ISDIC schemes is derived in

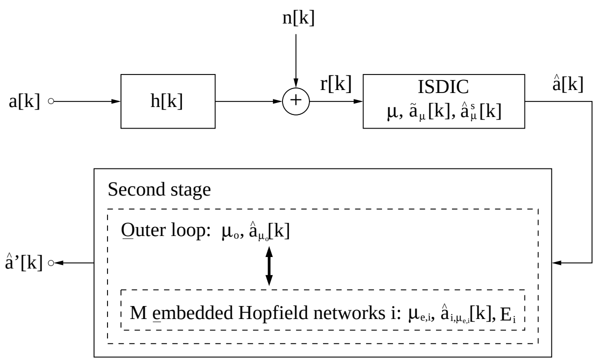

Fig. 1. Model of transmission.

Section 4. Simulation results for the novel algorithm are given in Section 5 for 4QAM and 16QAM. These are compared with the corresponding results for a DDFSE with various numbers of states and the matched filter bound.

2 System Model

An uncoded packet PAM (pulse amplitude modulation) transmission with QAM constellations and Gray mapping is considered. In the following, all signals are represented by their complex–valued baseband equivalents. The discrete–time received signal is given by

$$r[k] = \sum_{\kappa=0}^{q_h} h[\kappa]\, a[k-\kappa] + n[k], \qquad (1)$$

where $a[k] \in \mathcal{X} = \{x_1 = x_{I,1} + \mathrm{j}\, x_{Q,1}, \ldots, x_i = x_{I,i} + \mathrm{j}\, x_{Q,i}, \ldots, x_M = x_{I,M} + \mathrm{j}\, x_{Q,M}\}$ are the transmitted QAM coefficients at discrete time instants k which are taken from a set of cardinality M. The average power of the transmitted coefficients is denoted by $\sigma_a^2 = \mathcal{E}\{|a[k]|^2\}$. All coefficients $x_i = x_{I,i} + \mathrm{j}\, x_{Q,i}$ are assumed to be equally probable. The final estimate for $a[k]$ of the ISDIC stage is denoted by $\hat{a}[k]$ and $\hat{a}'[k]$ means the final estimate of the second stage. $h[\cdot]$ is the causal CIR of order q_h comprising transmit filter, channel, and continuous–time receiver input filter and $n[k]$ is additive complex white Gaussian noise with variance σ_n^2. We assume that the CIR is random but constant during the transmission of one data packet (block fading model) and that ideal channel state information is available at the receiver. In order to simplify notation, the discrete time index $k = 1$ is assigned to the first of L data symbols in each burst.

For the ISDIC algorithms we use a matrix vector notation, which is introduced in the following. First, we define a convolution matrix $\boldsymbol{H}$ of size $(L+q_h)\times L$ whose entries in the ith row and jth column are $\boldsymbol{H}_{(i,j)} = h[i-j]$. Then, the received signal vector $\boldsymbol{r} = [\,r[1], r[2], \ldots, r[L+q_h]\,]^T$ can be expressed as

$$\boldsymbol{r} = \boldsymbol{H} \cdot \boldsymbol{a} + \boldsymbol{n}\,, \qquad (2)$$

where the definitions $\boldsymbol{a} = [\,a[1], a[2], \ldots, a[L]\,]^T$ and $\boldsymbol{n} = [\,n[1], n[2], \ldots, n[L+q_h]\,]^T$ have been used. Furthermore, a truncated version of $\boldsymbol{r}$ is introduced for sliding window operations of the ISDIC algorithm. We consider $q_w \geq 0$ received samples before and $q_w + q_h$ received samples after a time instant k, respectively, for estimation of a certain symbol $a[k]$. Collecting these samples in a vector $\boldsymbol{r}_k$, we obtain

$$\boldsymbol{r}_k = \boldsymbol{H}_k \cdot \boldsymbol{a}_k + \boldsymbol{n}_k\,, \qquad (3)$$

with

$$\boldsymbol{r}_k = [\,r[k-q_w], \ldots, r[k], \ldots, r[k+q_h], \ldots, r[k+q_h+q_w]\,]^T\,, \qquad (4)$$

$\boldsymbol{H}_{k\,(i,j)} = h[q_h + i - j]$, where $\boldsymbol{H}_k$ has size $(2q_w + 1 + q_h)\times(2q_w + 1 + 2q_h)$, $\boldsymbol{a}_k = [\,a[k-q_w-q_h], \ldots, a[k], \ldots, a[k+q_w+q_h]\,]^T$, and $\boldsymbol{n}_k = [\,n[k-q_w], \ldots, n[k], \ldots, n[k+q_h], \ldots, n[k+q_h+q_w]\,]^T$. We assume that a sufficient number of zero tail symbols is sent prior and after each block, i.e., $a[k] = 0$ is valid for $k \notin \{1, 2, \ldots, L\}$. This means that e.g. $\boldsymbol{a}_1$ contains $q_w + q_h$ leading zeros and $\boldsymbol{a}_L$ contains $q_w + q_h$ trailing zeros.

3 Iterative Soft Decision Interference Cancellation (ISDIC)

As our proposed second stage for ISDIC algorithms in Section 4 is designed as a remedy to the shortcomings of the first stage, the basic principles of ISDIC are reviewed in the following. For more information the reader is referred to [1], [2], [3], [4], [13].

In each iteration of the ISDIC algorithm, soft–decision feedback is performed for cancellation of pre- and postcursor ISI in a sequential manner starting from $k = 1$ up to $k = L$. $\mu \in \{0, \ldots, \mu_{\mathrm{stop}}\}$ is the index of the current iteration and $\hat{a}_\mu^{\mathrm{s}}[k]$ denotes the soft decision on $a[k]$ calculated in iteration μ. The utilized soft decisions $\hat{a}_\mu^{\mathrm{s}}[k]$ minimize the mean–squared error after feedback in the current iteration under the assumption that the sum of noise and ISI can be modeled as Gaussian random variable with zero mean and uncorrelated inphase and quadrature components, cf. [14], [15]. The soft decisions $\hat{a}_\mu^{\mathrm{s}}[k]$ are initialized according to

$$\hat{a}_0^{\mathrm{s}}[k] = 0\,, \quad k \in \{1, 2, \ldots, L\}\,. \qquad (5)$$

For cancellation we introduce the vector

$$\hat{\hat{\boldsymbol{a}}}_{k,\mu}^{s} = [\,\hat{a}_\mu^s[k-q_w-q_h], \ldots, \hat{a}_\mu^s[k-1], 0, \hat{a}_{\mu-1}^s[k+1], \ldots, \hat{a}_{\mu-1}^s[k+q_w+q_h]\,]^T\,. \qquad (6)$$

In each iteration μ and for each $k \in \{1, 2, \ldots, L\}$ the following steps have to be done. In order to obtain an estimate for the desired coefficient $a[k]$, intersymbol interference from other coefficients is removed from the vector $\boldsymbol{r}_k$ in the best possible way using the latest soft estimates in vector $\hat{\tilde{\boldsymbol{a}}}^s_{k,\mu}$:

$$\begin{aligned} \boldsymbol{r}_{k,\mu} &= \boldsymbol{r}_k - \boldsymbol{H}_k \cdot \hat{\tilde{\boldsymbol{a}}}^s_{k,\mu} \\ &= \boldsymbol{H}_k \cdot \left(\boldsymbol{a}_k - \hat{\tilde{\boldsymbol{a}}}^s_{k,\mu}\right) + \boldsymbol{n}_k \, . \end{aligned} \quad (7)$$

This leads to a vector $\boldsymbol{r}_{k,\mu}$ which contains significantly less interference compared to $\boldsymbol{r}_k$ when the soft estimates contained in $\hat{\tilde{\boldsymbol{a}}}^s_{k,\mu}$ get better from iteration to iteration.

3.1 Matched Filter Based Iterative Soft Decision Interference Cancellation (MF ISDIC)

In the MF ISDIC the vector $\boldsymbol{r}_{k,\mu}$ is filtered by a vector $\boldsymbol{w}^H_{k,\mu}$ matched to the effective transmit pulse for symbol $a[k]$ which is equivalent to the CIR:

$$\boldsymbol{w}^H_{k,\mu} = \frac{1}{\rho_{k,k}} (\boldsymbol{H}_{k_{(:,q_w+q_h+1)}})^H \quad (8)$$

Here, $\boldsymbol{H}_{k_{(:,q_w+q_h+1)}}$ is the (q_w+q_h+1)th column of the matrix $\boldsymbol{H}_k$ and contains a shifted copy of the CIR for symbol $a[k]$. The term $\rho_{k,k}$ corresponds to the energy of the CIR and $(\cdot)^H$ denotes Hermitian transposition. The output of the matched filter is

$$\begin{aligned} \tilde{a}_\mu[k] &= \boldsymbol{w}^H_{k,\mu} \cdot \boldsymbol{r}_{k,\mu} = \frac{1}{\rho_{k,k}} (\boldsymbol{H}_{k_{(:,q_w+q_h+1)}})^H \cdot \boldsymbol{r}_{k,\mu} \\ &= a[k] + \sum_{\xi=k-q_w-q_h}^{k-1} \frac{\rho_{\xi,k}}{\rho_{k,k}} (a[\xi] - \hat{a}^s_\mu[\xi]) + \\ &\quad \sum_{\xi=k+1}^{k+q_w+q_h} \frac{\rho_{\xi,k}}{\rho_{k,k}} (a[\xi] - \hat{a}^s_{\mu-1}[\xi]) + \\ &\quad \frac{(\boldsymbol{H}_{k_{(:,q_w+q_h+1)}})^H}{\rho_{k,k}} \cdot \boldsymbol{n}_k \\ &= a[k] + n_{k,\mu} \end{aligned} \quad (9)$$

where we used the definition

$$[\, \rho_{k-q_w-q_h,k}, \ldots, \rho_{k,k}, \ldots, \rho_{k+q_w+q_h,k} \,] = (\boldsymbol{H}_{k_{(:,q_w+q_h+1)}})^H \cdot \boldsymbol{H}_k \quad (10)$$

and introduced the abbreviation $n_{k,\mu}$ for residual ISI and noise. The power $\sigma^2_{k,\mu}$ of $n_{k,\mu}$ is estimated exploiting conditioned expectations on the latest matched filter outputs for refinement of estimation as [13]

$$\begin{aligned} \hat{\sigma}^2_{k,\mu} &= \mathcal{E}\left\{ |n_{k,\mu}|^2 \middle| \tilde{\boldsymbol{a}}_{k,\mu} \right\} \\ &= \sum_{\xi=k-q_w-q_h}^{k-1} \frac{\rho^2_{\xi,k}}{\rho^2_{k,k}} \left(\mathcal{E}\{ |a[\xi]|^2 \,|\, \tilde{a}_\mu[\xi]\} - |\hat{a}^s_\mu[\xi]|^2 \right) + \\ &\quad \sum_{\xi=k+1}^{k+q_w+q_h} \frac{\rho^2_{\xi,k}}{\rho^2_{k,k}} \left(\mathcal{E}\{ |a[\xi]|^2 \,|\, \tilde{a}_{\mu-1}[\xi]\} - \right. \\ &\quad \left. |\hat{a}^s_{\mu-1}[\xi]|^2 \right) + \frac{\sigma^2_n}{\rho_{k,k}} \, , \end{aligned} \quad (11)$$

with

$$\begin{aligned} \tilde{\tilde{\boldsymbol{a}}}_{k,\mu} &= [\, \tilde{a}_\mu[k-q_w-q_h], \ldots, \tilde{a}_\mu[k-1], 0, \\ &\quad \tilde{a}_{\mu-1}[k+1], \ldots, \tilde{a}_{\mu-1}[k+q_w+q_h] \,]^T \, , \end{aligned} \quad (12)$$

where we used $\hat{a}^s_\mu[k] = \mathcal{E}\{a[k] | \tilde{a}_\mu[k]\}$ which minimizes the mean–squared error $\mathcal{E}\{|a[k] - \hat{a}^{\mathrm{s}}_\mu[k]|^2\}$, cf. [16].

The initialization of $\tilde{a}_\mu[k]$ is done according to:

$$\tilde{a}_0[k] = 0 \, , \quad \forall k. \quad (13)$$

We model residual ISI and noise $n_{k,\mu}$ as a random variable with a joint normal density [16] with zero mean and variance $\hat{\sigma}^2_{k,\mu}/2$ in real and imaginary part and zero correlation coefficient between them.

The soft estimate $\hat{a}^{\mathrm{s}}_\mu[k]$ for $a[k]$ can be derived for zero correlation coefficient as

$$\hat{a}^{\mathrm{s}}_\mu[k] = \mathcal{E}\{a[k] \,|\, \tilde{a}_\mu[k]\} = \frac{\sum_{i=1}^M x_i \, \mathrm{e}^{\left(-\frac{|\tilde{a}_\mu[k]-x_i|^2}{\hat{\sigma}^2_{k,\mu}}\right)}}{\sum_{i=1}^M \mathrm{e}^{\left(-\frac{|\tilde{a}_\mu[k]-x_i|^2}{\hat{\sigma}^2_{k,\mu}}\right)}} \quad (14)$$

($\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$: real part and imaginary part of complex numbers, respectively). The calculation of soft estimates for general complex–valued symbol alphabets according to Eq. (14) is also given in [17]. The advantage of using soft estimates in place of hard decisions in equalizers with feedback was first pointed out by Taylor in [18], where suboptimum estimates of the transmitted symbols have been used in the feedback section of a DFE.

For 4QAM transmission with $a[k] \in \{\pm 1 \pm \mathrm{j}\}$, Eq. (14) reads

$$\begin{aligned} \hat{a}^s_\mu[k] &= \tanh\left(\frac{2}{\hat{\sigma}^2_{k,\mu}} \mathrm{Re}\left\{\tilde{a}_\mu[k]\right\} \right) + \\ &\quad \mathrm{j} \tanh\left(\frac{2}{\hat{\sigma}^2_{k,\mu}} \mathrm{Im}\left\{\tilde{a}_\mu[k]\right\} \right), \end{aligned} \quad (15)$$

which was derived in [2].

After the current iteration has been finished a new iteration starts. The algorithm stops if soft decisions remain essentially unchanged from one iteration to the next, i.e.,

$$\begin{aligned} \max_{k\in\{1,2,\ldots,L\}} \left|\mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_\mu[k]\right\} - \mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_{\mu-1}[k]\right\}\right| &< \epsilon \quad \text{and} \\ \max_{k\in\{1,2,\ldots,L\}} \left|\mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_\mu[k]\right\} - \mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\mu-1}[k]\right\}\right| &< \epsilon \, , \end{aligned} \quad (16)$$

with a small constant ϵ, or the iteration number exceeds a prescribed limit $\mu_{\max}$.

Final hard estimates $\hat{a}[k]$ of the ISDIC stage for the data symbols are obtained by feeding the soft decisions of the last iteration μ_{stop} into a function $\mathcal{H}_M(\cdot)$:

$$\hat{a}[k] = \mathcal{H}_M\left(\hat{a}^s_{\mu_{\mathrm{stop}}}[k]\right) \quad k \in \{1, 2, \ldots, L\} \, , \quad (17)$$

where $\mathcal{H}_M(\cdot)$ denotes the characteristic of the slicer for the corresponding QAM constellation.

3.2 MMSE–Based Iterative Soft Decision Interference Cancellation (MMSE ISDIC)

Here, for calculation of an estimate $\tilde{a}_\mu[k]$ from the vector $\boldsymbol{r}_{k,\mu}$ an MMSE filter is used. Obviously, when calculating this MMSE filter, it is advantageous to utilize prior knowledge contained in the vector $\tilde{\tilde{\boldsymbol{a}}}_{k,\mu}$. Therefore, the MMSE filter $\boldsymbol{w}_{k,\mu}^H$ is determined based on conditioned expectations:

$$\boldsymbol{w}_{k,\mu}^H = \mathcal{E}\left\{a[k]\cdot\boldsymbol{r}_{k,\mu}^H|\tilde{\tilde{\boldsymbol{a}}}_{k,\mu}\right\}\cdot\left(\mathcal{E}\left\{\boldsymbol{r}_{k,\mu}\cdot\boldsymbol{r}_{k,\mu}^H|\tilde{\tilde{\boldsymbol{a}}}_{k,\mu}\right\}\right)^{-1}.$$

The vector $\tilde{\tilde{\boldsymbol{a}}}_{k,\mu}$, cf. Eq. (12), now contains the outputs of the MMSE filter after bias correction. Applying the derived MMSE filter $\boldsymbol{w}_{k,\mu}^H$ to the vector $\boldsymbol{r}_{k,\mu}$ we get

$$\boldsymbol{w}_{k,\mu}^H\boldsymbol{r}_{k,\mu} = b_{k,\mu}a[k] + i_{k,\mu}\,, \tag{18}$$

with a bias $b_{k,\mu}$ produced by the MMSE filter and a distortion $i_{k,\mu}$ composed of the sum of residual interference and channel noise. The bias term can be expressed as

$$b_{k,\mu} = \boldsymbol{w}_{k,\mu}^H\boldsymbol{H}_{k_{(:,q_w+q_h+1)}}\,. \tag{19}$$

Multiplying the filter output with a factor $1/b_{k,\mu}$ we get the unbiased signal:

$$\tilde{a}_\mu[k] = \frac{\boldsymbol{w}_{k,\mu}^H\boldsymbol{r}_{k,\mu}}{b_{k,\mu}} = a[k] + \frac{i_{k,\mu}}{b_{k,\mu}}\,. \tag{20}$$

The initialization of $\tilde{a}_\mu[k]$ is done according to Eq. (13). As can be seen from the definition of the truncated vectors in Section 2, the resulting length of the MMSE filter is $2q_w + 1 + q_h$.

Using results of [19] we obtain an estimate of the power $\hat{\sigma}_{k,\mu}^2 \approx \mathcal{E}\{|i_{k,\mu}/b_{k,\mu}|^2\}$ of residual interference and noise after bias compensation effective for symbol $a[k]$ in iteration μ:

$$\hat{\sigma}_{k,\mu}^2 = \sigma_a^2\,\frac{1-b_{k,\mu}}{b_{k,\mu}}\,. \tag{21}$$

The probability density function of residual interference and noise after bias compensation is again assumed to be a joint normal density [16] with zero mean and variance $\hat{\sigma}_{k,\mu}^2/2$ in real and imaginary part and zero correlation coefficient between them.

Soft estimates are determined according to Eqs. (14), (15). The algorithm stops according to the criterion in Eq. (16) and hard estimates $\hat{a}[k]$ are calculated thereafter.

4 Stage for Error Search and Correction

4.1 Analysis of errors occurring in ISDIC

As we will show in Section 5 an error floor occurs especially for the MF ISDIC at high signal–to–noise ratios (SNRs). Having a detailed look at these errors one recognizes that error patterns exist, which lock themselves and are stable during MF ISDIC iterations.

This means, given some errors in the values $\hat{a}_\mu^s[k]$ it is possible that cancellation using erroneously detected symbols causes an amount of interference that causes other symbols to be erroneously detected which in turn cause the first group of symbols to be detected in the same wrong way. We want to denote these wrong detected symbols as locked error pattern.

For the following analysis we consider first

$$\sigma_n^2 = 0 \tag{22}$$

as the locked error patterns we want to consider also occur for very high SNR and assume that

$$\hat{a}_{\mu_{\text{stop}}}^s[k] \in \mathcal{X} \quad \forall k\,. \tag{23}$$

Both assumptions lead to $\hat{\sigma}_{k,\mu}^2 = 0$ according to Eq. (11) and result in the fact that only hard decisions are performed. Given a vector

$$\hat{\boldsymbol{a}}_{\mu_{\text{stop}}}^s = [\,\hat{a}_{\mu_{\text{stop}}}^s[1], \hat{a}_{\mu_{\text{stop}}}^s[2], \ldots, \hat{a}_{\mu_{\text{stop}}}^s[L]\,]^T \tag{24}$$

that contains such locked error patterns, i.e., $\hat{\boldsymbol{a}}_{\mu_{\text{stop}}}^s \neq \boldsymbol{a}$,

$$\hat{\boldsymbol{a}}_{\mu_{\text{stop}}}^s = \mathcal{H}_M'\left(\frac{1}{\rho_{1,1}}\left(\boldsymbol{H}^H\boldsymbol{H}\cdot\boldsymbol{a} - (\boldsymbol{H}^H\boldsymbol{H} - \boldsymbol{I}\rho_{1,1})\cdot\hat{\boldsymbol{a}}_{\mu_{\text{stop}}}^s\right)\right) \tag{25}$$

holds for the MF ISDIC where Eqs. (2) and (8) have been used. Here, we utilized the generalization of the hard decision function of Eq. (17) to vectors which is denoted as $\mathcal{H}_M'(\cdot)$. For simplicity we used $\rho_{1,1}$ in Eq. (25) and $\boldsymbol{I}$ is the identity matrix.

Obviously, the matrix $\boldsymbol{H}$ and therefore the CIR has major impact on the occurrence of locked error patterns $\hat{\boldsymbol{a}}_{\mu_{\text{stop}}}^s \neq \boldsymbol{a}$. Eq. (25) emphasizes the reason for the error floor of the MF ISDIC mentioned above. Obviously, locked error patterns are stable during MF ISDIC iterations, i.e., they are fix points of the function on the right hand side of Eq. (25).

Having understood the reason for the error floor of the MF ISDIC it is possible to relax the restrictions Eqs. (22) and (23) leading to a hardly tractable relation for locked error patterns $\hat{\boldsymbol{a}}_{\mu_{\text{stop}}}^s \neq \boldsymbol{a}$ instead of Eq. (25) as Eqs. (11) and (14) have to be incorporated. But it is easily understood that also in this case locked error patterns occur based on the same effect as described in Eq. (25). And even when applying the MMSE ISDIC errors based on the same effect may occur.

4.2 Remedy and proposed second stage for ISDIC

A major conclusion that can be drawn from the previous subsection is that a locked error pattern $\hat{\boldsymbol{a}}_{\mu_{\text{stop}}}^s \neq \boldsymbol{a}$ is stable but would decompose at least partly in course of further ISDIC iterations if one of the erroneous symbols is set to its correct value as the interference coming up from cancelling using a wrong estimate is not existent any more for estimating other symbols erroneously. In other words, the error pattern would become unstable during further ISDIC iterations.

A second very useful insight is gained from Hopfield network theory [10], [11], [12]. It can be shown that a Hopfield network applied to the given communications problem converges during several iterations $\breve{\mu}$ to a minimum of

$$E = ||\boldsymbol{r} - \boldsymbol{H} \cdot \breve{\boldsymbol{a}}_{\breve{\mu}}||^2 \qquad (26)$$

($||\cdot||$: L_2 norm of a vector) with $\breve{\boldsymbol{a}}_{\breve{\mu}} = [\,\breve{a}_{\breve{\mu}}[1], \ldots, \breve{a}_{\breve{\mu}}[L]\,]$, $\breve{a}_{\breve{\mu}}[k] \in \mathcal{X}$, but not necessarily to the global one when updating $\breve{\boldsymbol{a}}_{\breve{\mu}}$ according to:

$$\breve{a}_{\breve{\mu}}[k] = \mathcal{H}_M \left(\frac{1}{\rho_{k,k}} \boldsymbol{H}^H_{(:,k)} \left(\boldsymbol{r} - \boldsymbol{H} \cdot \breve{\breve{\boldsymbol{a}}}_{\breve{\mu}} \right) \right) \qquad (27)$$

for all $k \in \{1, \ldots, L\}$ in each iteration $\breve{\mu}$ with $\breve{\breve{\boldsymbol{a}}}_{\breve{\mu}} = [\,\breve{a}_{\breve{\mu}}[1], \ldots, \breve{a}_{\breve{\mu}}[k-1], 0, \breve{a}_{\breve{\mu}-1}[k+1], \ldots, \breve{a}_{\breve{\mu}-1}[L]\,]$. Obviously, replacing the soft decision according to Eq. (14) in the MF ISDIC with a hard decision function $\mathcal{H}_M(\cdot)$ exactly leads to the described Hopfield network which would perform worse than the MF ISDIC. In contrast to the described Hopfield network an MLSE would find the global minimum of the function in Eq. (26) but at a very high effort. From this point of view, the locked error patterns correspond to local minima of E.

Taking all previously mentioned results into account we can propose a second stage for error search and correction for ISDIC algorithms based on Hopfield network theory. The proposed second stage according to Fig. 1 consists of an outer loop and embedded Hopfield networks and is described in the following.

4.2.1 Outer Loop

The second stage optimizes all estimates $\hat{a}[k]$ from the ISDIC stage in several iterations μ_o. The vector $\hat{\boldsymbol{a}}_{\mu_o}$ contains the latest estimates of the outer loop in iteration μ_o:

$$\hat{\boldsymbol{a}}_{\mu_o} = [\,\hat{a}_{\mu_o}[1], \ldots, \hat{a}_{\mu_o}[k], \ldots, \hat{a}_{\mu_o}[L]\,]\,. \qquad (28)$$

$\hat{\boldsymbol{a}}_{\mu_o}$ is initialized with the final hard estimates (cf. Eq. (17)) from the ISDIC stage:

$$\hat{\boldsymbol{a}}_0 = [\,\hat{a}[1], \ldots, \hat{a}[k], \ldots, \hat{a}[L]\,]\,. \qquad (29)$$

M embedded Hopfield networks are initiated in each iteration μ_o of the outer loop and for each symbol k_o starting from $k_o = 1$ to $k_o = L$ leading to M different estimates for the vector $\boldsymbol{a}$. The best of these M estimates is stored in the vector $\hat{\boldsymbol{a}}_{\mu_o}$.

The algorithm stops if the latest estimates in the outer loop remain unchanged from one iteration μ_o to the next, i.e., $\hat{\boldsymbol{a}}_{\mu_o+1} = \hat{\boldsymbol{a}}_{\mu_o}$ yielding the final estimates of our proposed second stage in vector $\hat{\boldsymbol{a}}_{\mu_o+1}$. The output of the second stage is then determined according to $\hat{a}'[k] = \hat{a}_{\mu_o+1}[k]\ \forall k \in \{1, \ldots, L\}$.

4.2.2 Embedded Hopfield networks

In each iteration μ_o of the outer loop and for each symbol k_o we build M vectors $\hat{\boldsymbol{a}}_{i,0}$ with $i \in \{1, \ldots, M\}$ from the latest estimates $\hat{a}_{\mu_o}[k]$ of the outer loop as an initialization for M embedded and independent Hopfield networks:

$$\begin{aligned} \hat{\boldsymbol{a}}_{i,0} &= [\,\hat{a}_{i,0}[1], \ldots, \hat{a}_{i,0}[L]\,] \qquad (30)\\ &= [\,\hat{a}_{\mu_o}[1], \ldots, \hat{a}_{\mu_o}[k_o-1], x_i, \\ &\qquad \hat{a}_{\mu_o}[k_o+1], \ldots, \hat{a}_{\mu_o}[L]\,] \end{aligned}$$

Each embedded Hopfield network i calculates estimates similarly to Eq. (27) in each iteration $\mu_{e,i}$:

$$\hat{a}_{i,\mu_{e,i}}[k] = \mathcal{H}_M \left(\frac{1}{\rho_{k,k}} \boldsymbol{H}^H_{(:,k)} \left(\boldsymbol{r} - \boldsymbol{H} \cdot \hat{\hat{\boldsymbol{a}}}_{i,\mu_{e,i}} \right) \right) \qquad (31)$$

for all $k \in \{1, \ldots, k_o - 1, k_o + 1, \ldots, L\}$ in each iteration $\mu_{e,i}$ with $\hat{\hat{\boldsymbol{a}}}_{i,\mu_{e,i}} = [\,\hat{a}_{i,\mu_{e,i}}[1], \ldots, \hat{a}_{i,\mu_{e,i}}[k-1], 0, \hat{a}_{i,\mu_{e,i}-1}[k+1], \ldots, \hat{a}_{i,\mu_{e,i}-1}[L]\,]$. Assumed, the latest estimate $\hat{a}_{\mu_o}[k_o]$ is erroneous and belongs to a locked error pattern, then, as one of the x_i with $i \in \{1, \ldots, M\}$ set in Eq. (30) corresponds to $a[k_o]$, preventing an update at time instant k_o ensures a decomposition of the locked error pattern at least partly for one of the M Hopfield networks.

Each of the M embedded Hopfield networks stops if $\hat{a}_{i,\mu_{e,i}-1}[k] = \hat{a}_{i,\mu_{e,i}}[k]\,\forall\, k \in \{1, \ldots, k_o - 1, k_o + 1, \ldots, L\}$. The corresponding iteration may be denoted with $\mu_{e,\text{stop},i}$. For each embedded Hopfield network Eq. (26) can be evaluated leading to:

$$E_i = ||\boldsymbol{r} - \boldsymbol{H} \cdot \hat{\boldsymbol{a}}_{i,\mu_{e,\text{stop},i}}||^2 \qquad (32)$$

with $\hat{\boldsymbol{a}}_{i,\mu_{e,\text{stop},i}} = [\,\hat{a}_{i,\mu_{e,\text{stop},i}}[1], \ldots, \hat{a}_{i,\mu_{e,\text{stop},i}}[k_o-1], x_i, \hat{a}_{i,\mu_{e,\text{stop},i}}[k_o+1], \ldots, \hat{a}_{i,\mu_{e,\text{stop},i}}[L]\,]$.

Given that i_0 yields the minimum squared norm

$$E_{i_0} = \min_{i \in \{1,\ldots,M\}} E_i\,, \qquad (33)$$

the estimates in the outer loop are updated according to

$$\hat{a}_{\mu_o}[k] \;=\; \begin{cases} \hat{a}_{i_0,\mu_{e,\text{stop},i_0}}[k] & \forall\, k \in \{1, \ldots, L\} \backslash k_o \\ x_{i_0} & \text{for} \quad k = k_o \end{cases}$$

and used for the subsequent Hopfield networks started by the outer loop for time–instant $k_o + 1$ if $k_o < L$ or for the first symbol in the next iteration if $k_o = L$.

4.2.3 Remarks

In contrast to MLSE where the global minimum of E in Eq. (26) is found by a high complexity combinatorial search, we utilize the facts that a simple Hopfield network always finds a local minimum of Eq. (26) and that the corresponding locked error patterns decompose at least partly in course of Hopfield iterations if one of the erroneously detected symbols is set to its correct value what can be recognized by evaluating Eq. (26). The proposed algorithm moves on the surface of E from one local minimum downwards to the next one towards the global one.

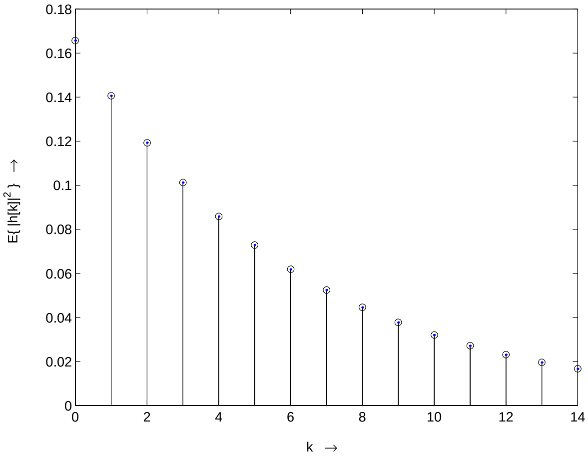

Fig. 2. Power delay profile of channel A ($q_h = 14$).

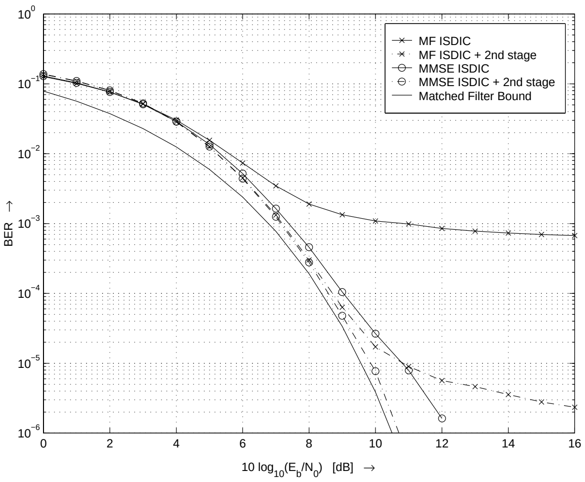

Fig. 3. BER vs. $E_b/\mathcal{N}_0$ for channel A with optimized ISDIC channel equalization for 4QAM transmission, ideal power control.

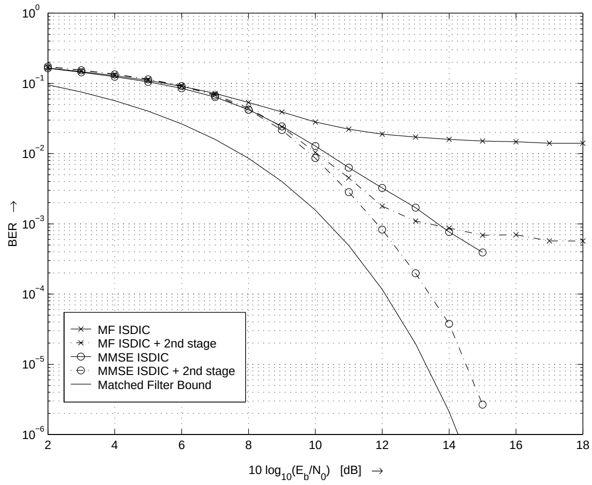

Fig. 4. BER vs. $E_b/\mathcal{N}_0$ for channel A with optimized ISDIC channel equalization for 16QAM transmission, ideal power control.

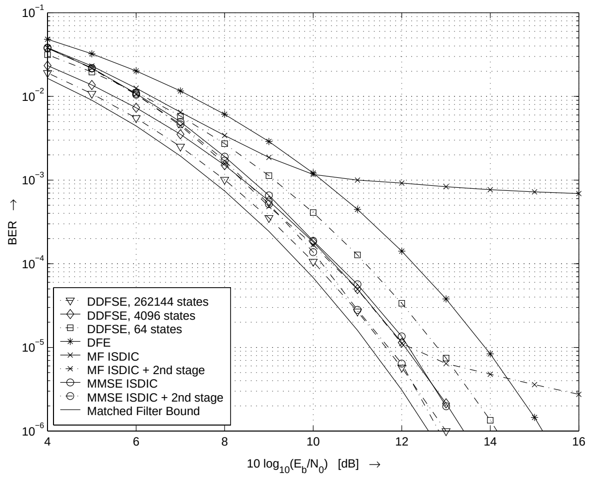

Fig. 5. BER vs. $E_b/\mathcal{N}_0$ for channel A with optimized ISDIC channel equalization for 4QAM transmission, without power control.

5 Numerical Results and Discussion

In the following, the performance of the proposed algorithm is analyzed for transmission over three different Rayleigh fading channels A, B, and C. Channel A consists of 15 taps with exponentially decreasing average power. The corresponding power delay profile is given in Fig. 2. As a kind of worst case scenario the power delay profile of channel B consists of 10 taps with equal power whereas the power delay profile of channel C has 20 taps with equal power. In each case the channel taps are complex–valued and time–invariant within a frame (block fading). For all three channels we assume ideal power control in Figs. 3, 4, 6, and 7, which can be understood as normalizing

$$E_{\mathrm{CIR}} = \sum_{k=0}^{q_h} |h[k]|^2 \tag{34}$$

to unity. Fig. 5 gives the performance of the proposed algorithm for channel A without power control.

For simulations, data frame length has been chosen to $L = 3 \cdot 256$, number of iterations maximally tolerated to $\mu_{\max} = 40$, and $\epsilon = 1 \cdot 10^{-2}$ at an average symbol power of $\sigma_a^2 = 4$ for 16QAM and $\sigma_a^2 = 2$ for 4QAM. We used $q_w = 5$ in the MMSE–based algorithm in order to have a sufficient filter length for interference suppression.

Figs. 3 to 7 show the performance of the proposed second stage with MMSE ISDIC and MF ISDIC, respectively, as first stage. Also the performance of the first stage is given to show the additional gain of the proposed second stage. Due to the ideal power control the matched filter bounds in Figs. 3, 4, 6, and 7 are simply given by the AWGN channel performance for 4QAM and 16QAM, respectively.

Figs. 3 and 4 show the performance of MF ISDIC and MMSE ISDIC utilizing the proposed second stage for 4QAM and 16QAM transmission over channel A. A remarkable result is that the MF ISDIC with second

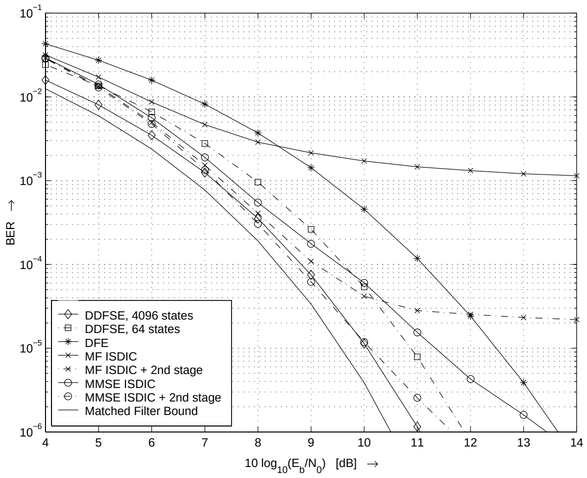

Fig. 6. BER vs. $E_b/\mathcal{N}_0$ for channel B with optimized ISDIC channel equalization for 4QAM transmission, ideal power control.

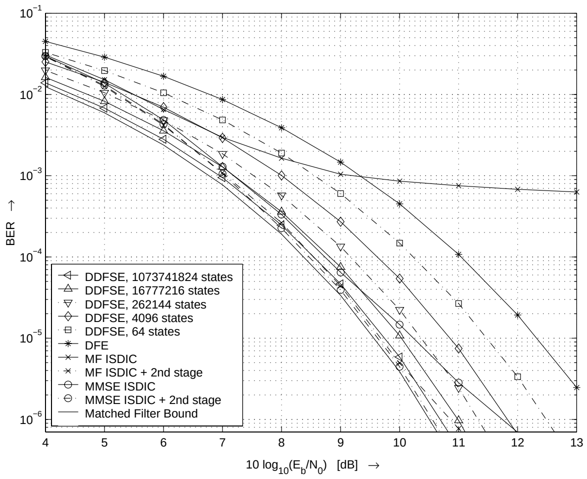

Fig. 7. BER vs. $E_b/\mathcal{N}_0$ for channel C with optimized ISDIC channel equalization for 4QAM transmission, ideal power control.

stage outperforms the more complex MMSE ISDIC without second stage in an SNR range up to about 11 dB for 4QAM and up to about 14 dB for 16QAM. In the latter case the additional gain is 1 dB. The error floor of the MF ISDIC can be lowered by utilizing the proposed second stage by a factor of about 100 for 4QAM and about 20 for 16QAM. For the MMSE ISDIC we gain about 1 dB for 4QAM and about 2 dB for 16QAM by utilizing the proposed second stage and reach the respective matched filter bound up to a fraction of a dB for 4QAM and up to 1 dB for 16QAM.

As can be seen from Figs. 3 and 4 the performance of our proposed scheme is very close to the matched filter bound and the question arises, whether other implementable receiver algorithms can achieve a better performance or not. Obviously, MLSE is by far too complex for the adopted channels. For this reason, we consider DDFSE with various numbers of states. The comparison of our proposed scheme and DDFSE performance bounds for 4QAM transmission for channels A, B, and C is given in Figs. 5, 6, and 7. The performance bounds for DDFSE are calculated by averaging

$$\mathrm{Q}\left(\sqrt{\frac{E_{\mathrm{CIR}}}{\mathcal{E}\{E_{\mathrm{CIR}}\}}\, d^2_{\min} \frac{E_b}{\mathcal{N}_0}}\right), \qquad (35)$$

where the normalized squared Euclidean distance $d^2_{\min}$ of DDFSE has been determined for each channel realization using the Dijkstra algorithm [20], [21]. For each channel realization an allpass filter concentrating the energy of the CIR in the first taps was applied in front of equalization.

Results for channel A without power control and 4QAM transmission are given in Fig. 5. A comparison with DDFSE shows that MF ISDIC with second stage and MMSE ISDIC perform similar to a DDFSE with 4096 states and MMSE ISDIC with second stage approaches the performance of DDFSE with 262144 states for high SNR.

Fig. 6 shows that MMSE ISDIC with second stage outperforms DDFSE with 4096 states in the range of 7 dB to 10 dB for channel B. Already in [22], where MF ISDIC has been proposed for CDMA multiuser equalization it has been observed that the performance of the algorithm improves when the spreading factor rises. Analogous behavior has been stated in [1] where it has been reported that the MF ISDIC for channel equalization outperforms DDFSE with a high number of states for long CIRs, e.g. of length 60, but for moderate length an error floor occurs. Similar to these observations we recognize that the performance of MF ISDIC and MMSE ISDIC improves for channel C in Fig. 7 which has double length of channel B. From Fig. 7 it can be seen that the MF ISDIC with second stage and the MMSE ISDIC with second stage approach DDFSE with about 10^9 states and reach the matched filter bound up to a fraction of a dB. The MMSE ISDIC performs similar to DDFSE with 16777216 states in an SNR range from about 7 dB to 9 dB. The most interesting result is that the MF ISDIC with second stage has about the same performance than MMSE ISDIC with second stage and outperforms the MMSE ISDIC although it has dramatically less complexity as no matrix inversions have to be performed each symbol in each iteration. This means that an MF ISDIC with second stage can already reach the matched filter bound for channels with moderate length whereas a single MF ISDIC needs very long CIRs, e.g. a length of 60 [1].

In summary, a MF ISDIC with subsequent second stage requiring negligible memory, where the second stage is only a modified hard decision interference cancellation scheme based on MF outputs, can perform as good as a DDFSE with about 10^9 states which is currently not implementable at all. In contrast, for longer CIRs MLSE and high–performance DDFSE have an exponentially increasing complexity and are therefore not implementable. As shown in Figs. 6 and 7 and as can be concluded from [22], [1] the performance of the ISDIC schemes with second stage improves as the CIR gets longer and already approaches the matched filter bound up to a fraction of a dB for channel C with length 20.

6 Concluding Remarks

For MF ISDIC and MMSE ISDIC applied to channel equalization a second stage for error search and correction has been introduced and analyzed. Depending on channel and modulation format the performance improves by up to 1 dB to 2 dB for an MMSE ISDIC as first stage. The performance of MMSE ISDIC can be reached or even surpassed by the MF ISDIC with second stage requiring much less complexity. For the investigated channels the ISDIC scheme with following second stage approaches the matched filter bound up to a fraction of a dB for 4QAM and up to 1 dB for 16QAM. For channels of moderate length, e.g. 20 symbol–spaced taps, even the low–complexity MF ISDIC with second stage approaches the performance of a DDFSE with 10^9 states. It has been shown that the proposed ISDIC scheme with second stage performs better for longer CIRs preserving its moderate complexity whereas MLSE and high–performance DDFSE have an exponentially increasing complexity and are therefore not implementable for long CIRs. In this paper, a block–oriented transmission has been assumed, but the algorithms can also be accommodated to continuous transmission, cf. [1]. Additionally, it should be emphasized that the proposed second stage can also be utilized for an ISDIC algorithm applied to multiuser detection for CDMA transmission. A more comprehensive analysis of the proposed algorithm applied on PAM and CDMA transmission will be presented in [23].

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