DISTANCE-GAINS BY MULTIPLE-DUPLEX TRANSMISSION, CODING, AND SHAPING FOR HDSL

Johannes Huber

Lehrstuhl fuer Nachrichtentechnik,
Universitaet Erlangen-Nuernberg,
Cauerstrasse 7/NT, W-8520 Erlangen, Germany

ABSTRACT The maximum distance over which reliable high rate digital transmission over twisted pair lines is possible, can substantially be increased by multiple-duplex transmission, channel coding, and signal shaping. Simple equations are derived to transform SNR-gains into distance gains. Numerical examples are given for a transmission of 2.048 Mbit/s over twisted pairs with lines diameters 0.4 mm and 0.6 mm (AWG 26 and AWG 22). A new signal shaping method without scrambling of data bits is proposed, by which a SNR-loss due to error multiplication in the descrambling procedure is avoided.

1. INTRODUCTION

High-rate digital subscriber-lines (HDSL) are proposed to establish a "copper brigde" to a future optical broadband communication network, [?]. This technique offers a fast and cost efficient introduction of high rate digital communication services by the use of old simple twisted pair lines. System proposals with single-, dual- (1.544 Mbit/s) and even triple-duplex transmission ($\geq$ 2.048 Mbit/s) are discussed, cf. [?]. The goal is to attain to as many participants as possible without intermediate signal regeneration and without a selection of special pairs. The maximum distance over which reliable transmission is possible, can substantially be increased by multiple-duplex transmission, and by the application of channel coding and signal shaping. In this paper simple equations for an approximative calculation of such distance gains are derived.

For time-invariant channels which cause severe intersymbol interference, precoding, i.e. preequalization at the transmitter combined with a minimization of the signal power, is well suited for equalization, especially when channel coding is applied, [?,?]. The signal to noise ratio $\mathrm{SNR}_{\mathrm{prec}}$ for such schemes can be calculated via the $\mathrm{SNR}_{\mathrm{DFE}}$ of the equivalent structure with decision feedback equalization (DFE) and a zero-forcing whitened-matched-filter at the receiver input, cf. [?,?]. Examples for the transformation of coding and shaping gains into distance gains are given for single- to triple-duplex transmission of 2.048 Mbit/s over twisted pairs with line diameters 0.4 mm and 0.6 mm (AWG 26 and AWG 22). A simple new method for signal shaping without scrambling of data bits is proposed, which avoids SNR-losses due to error propagation in a descrambling process. The results show that the distance of HSDL can be more than doubled by multiple duplex transmission, coding and shaping.

2. APPROXIMATIVE SNR CALCULATION

The spectral attenuation $a(f)$ per km of a twisted pair line can be expressed by a sum of a few powers of the normalized frequency with sufficient accuracy, [?]:

$$a(f) = \sum_{i=0}^{n-1} a_i \left(\frac{f}{f_0}\right)^{b_i} \quad ; \quad a_i, b_i \in \mathrm{IR}\,; \tag{1}$$

f_0 : normalization frequency .

Examples:

(i) Signal distortions, which essentially are caused by skin effect (very high rate applications) are characterized by $n = 1$; $b_0 = 0.5$.

(ii) For the numerical results in this paper, the approximations of [?] are used to characterize twisted pair lines:
diameter $0.4\,\mathrm{mm}(\mathrm{AWG26}) : n = 2; f_0 = 1\,\mathrm{MHz};$
$a_0 = 4.5\,\mathrm{dB/km}; b_0 = 0; a_1 = 19\,\mathrm{dB/km}\,; b_1 = 0.7$
diameter $0.6\,\mathrm{mm}(\mathrm{AWG22}) : n = 2; f_0 = 1\,\mathrm{MHz};$
$a_0 = 2.2\,\mathrm{dB/km}; b_0 = 0; a_1 = 13\,\mathrm{dB/km}\,; b_1 = 0.65$

(iii) A Gaussian low pass filter corresponds to $n = 1;\ b_0 = 2$.

In a multipair cable noise is mainly caused by crosstalk of similar signals in adjacent pairs. Near end crosstalk (NEXT) is dominant over far end crosstalk (FEXT) and thermal noise. For sufficiently long lines, the NEXT transfer function $H_X(f)$ is independent of the cable length ℓ and can be approximated by

$$|H_X(f)|^2 = k_X (f/f_0)^X\ . \tag{2}$$

The constant k_X comprises the number of interfering signals and phase difference between the cyclic stationary crosstalk noise and the sampling clock in the receiver. For numerical results the usual value $X = 1.5$ is used. (15 dB decrease/decade, k_X needs not to be specified here.)

A baseband M-level transmission of R bit per symbol and twisted pair line is assumed for schemes which use I

pairs per system (I-fold-duplex-transmission). The symbol rate per pair is

$$\frac{1}{T} = \frac{\phi}{IR} \text{ with } \phi = 2.048 \cdot 10^6 \frac{\text{bit}}{\text{s}}\,. \tag{3}$$

According to [?,?], the signal to noise ratio SNR_{DFE} at the output of an optimum, minimum-phase, whitened-matched receiver input-filter and a subsequent error-free decision-feedback-equalization can be expressed in terms of the spectral signal to noise ratio $\text{SNR}_R(f)$ at the receiver input:

$$\text{SNR}_{\text{DFE}} = \exp\left\{ T \int_{-\frac{1}{2T}}^{+\frac{1}{2T}} \ln\left(\sum_{i=-\infty}^{\infty} \text{SNR}_R(f - i/T) \right) \mathrm{d}f\,. \right. \tag{4}$$

For a crosstalk dominated environment, $\text{SNR}_R(f)$ is independent of the transmitter pulse and given by

$$\text{SNR}_R(f) = 10^{-\ell a(f)/10} / |H_X(f)|^2\,. \tag{5}$$

In eq. (??) only such slots $\text{SNR}(f - i/T)$ of the folded spectral signal to noise ratio contribute to the sum, for which the spectral power density of the transmitter output signal is unequal zero. Therefore, in contrast to the usual power limitation and interference by white noise, the SNR_{DFE} is maximized by transmitter pulses with infinite bandwidth, e.g. rectangular pulses, for crosstalk.

Channel coding cannot be applied to systems with a DFE-receiver in a simple way, because DFE needs decisions without delay. (This problem may be eluded by interleaving.) This problem is avoided by the equivalent structure with Tomlinson-Harashima-precoding, cf. [?]. For the combination of Tomlinson-Harashima-precoding with a signal shaping procedure [?] and M level signalling, the signal to noise ratio for precoding is approximately given by:

$$\text{SNR}_{\text{prec}} = \frac{M^2 - 1}{M^2} \cdot 10^{G_s/10} \cdot \text{SNR}_{\text{DFE}}\,. \tag{6}$$

The first factor is caused by the almost continuous distribution of the signal levels at the output of the Tomlinson-Harashima-precoder. The second factor regards to a net shaping gain G_s.

By omitting the smaller terms $\text{SNR}_R(f-i/T) \ll \text{SNR}_R(f)$; $i \neq 0$, we get a simple lower bound of (4) in a crosstalk environment:

$$\begin{aligned} 10\log_{10}(\text{SNR}_{\text{DFE}}) \geq\; & 10\ell \cdot T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \log_{10}(\text{SNR}_R(f))\mathrm{d}f \\ =\; & -\ell \cdot A\left(\frac{\phi/f_0}{2IR}\right) - 10\log_{10}(k_X) \\ & -10X\log_{10}\left(\frac{\phi/f_0}{2IR}\right) + 10X\log_{10}(\mathrm{e}) \end{aligned} \tag{7}$$

where

$$A(x) = \frac{1}{x}\sum_{i=0}^{n-1} a_i \frac{x^{b_i+1}}{1+b_i} = \sum_{i=0}^{n-1} a_i \frac{x^{b_i}}{1+b_i} \tag{8}$$

is a normalized integral function of the line attenuation per km. Fig. 1 shows the difference between the (4) and (7) versus the cable length. The result proves that (7) is a sufficiently accurate approximation for field lengths which are interesting in practice.

3. DISTANCE GAIN BY MULTIPLE-DUPLEX TRANSMISSION

From (7) the maximum distance ℓ_I using I pairs can easily be expressed in terms of the maximum distance ℓ_1 using one pair, equal SNR_{DFE} assumed in both cases:

$$\ell_I A\left(\frac{\phi/f_0}{2IR}\right) = \ell_1 A\left(\frac{\phi/f_0}{2R}\right) + 10(X-1)\log_{10}(I) \tag{9}$$

In this formula we assume that, compared to the situation using one pair, the number of NEXT producing signals is multiplied by I for I pairs per subscriber. This factor implies an ideal cancellation of self NEXT within one system. This assumption is optimistic for multiple-duplex transmission because many pairs with low mutual crosstalk attenuation have to be used which could be avoided by a single duplex transmission. Fig. 2 shows maximum distances for reliable transmission over I pairs. The curves start at 2.7 km (AWG 26) and 4 km (AWG 22), as today avaluable quaternary systems with DFE for the transmission of 2.048 Mbit/s are specified for these distances, see [?].

4. DISTANCE GAIN BY CODING

A coding gain makes possible to lower the minimum acceptable SNR_{DFE} and by this an increase of the maximum distance. The coding gain is composed of a pure coding gain G_c, i.e. increase of minimum Euclidean distance, and a rate gain (loss) as usual. Assuming equal reliability as for an uncoded reference scheme with length ℓ_u, rate R_u, and I-fold transmission, the maximum length ℓ_c of the coded scheme with rate R_c and I-fold transmission is given by

$$\ell_c = \frac{\ell_u \cdot A\left(\frac{\phi/f_0}{2IR_u}\right) + 10X\log_{10}(R_c/R_u)}{A\left(\frac{\phi/f_0}{2IR_c}\right)} + \frac{G_c}{A\left(\frac{\phi/f_0}{2IR_c}\right)} \tag{10}$$

Fig. 3 shows distance gains $\ell_c - \ell_u$ for the ASK and QASK trellis-codes, which are listed in [?], over quaternary single- to triple-duplex transmission ($I = 1, 2, 3$) versus the number 2^ν of states of the trellis-encoder. For ASK-codes the one dimensional 8-ary constellation ($D = 1$; $M = 8$) and for QASK-Codes a two-dimensional constellation with 32 points similar to the proposal in [?] is assumed, $D = 2$: $M = \sqrt{32}$, cf. fig. 4. For these examples, the rate of $R_c =$ 2 bit per symbol and pair is the same as for the uncoded reference schemes. Therefore, there is no rate loss and the asymptotic distance gain can approximately be calculated from the free Euclidean distance d_{free} and the number N_{free}

of nearest neighbour error events of the trellis-code by

$$\ell_c - \ell_u \approx \left(20\log_{10}\left(\frac{d_{\text{free}}}{d_0}\right) - \frac{20}{D}\log_{10}(2) - \frac{2}{3}\log_{10}(N_{\text{free}}/2)\right) \Big/ A\left(\frac{\phi/f_0}{4I}\right) \quad (11)$$

Notice that the loss due to signal constellation expansion for Tomlinson-Harashima-precoding equals to the asymptotic loss for $M \to \infty$ without precoding (cf. [?]) for any M. By this effect, the power loss $(M^2-1)/M^2$ (cf. (??)) of precoding is compensated. In (11), the rule of thumb is applied, that the coding gain is decreased by 0.2 dB per doubling the number of nearest neighbour error events. Fig. 3 shows that in all cases one-dimensional trellis-codes are superior to two-dimensional.

5. DISTANCE GAIN BY SIGNAL SHAPING

Channels with noise caused by crosstalk of similar signals may be quoted as "classical" for the application of signal shaping in order to reduce average signal power because signal power simultaneously is noise power for other pair lines within a multipair cable. (In almost all other cases a peak power constraint is more relevant in practice than an average power constraint due to saturation effects of power amplifiers.) Unfortunately a great number of trellis-states is necessary to gain a noticable amount of signal power for twisted pair lines with severe intersymbol interference when trellis-precoding [?] is applied. Fig. 5 shows gross shaping gains (simulation results) for several shaping-methods combined with precoding for a DC free channel response with length 11 ($M=4$, $\ell = 2.7$ km, AWG 26). Trellis-precoding needs a descrambler at the receiver in order to extract the message from the sequence of detected (decoded) signal levels. Even a noncatastrophic descrambler causes a multiplication of symbol errors by error propagation which should not be neglected. The error multiplier increases with the number of shaping-encoder states. Therefore, the net shaping gain G_s including the loss due to error propagation is about 0.65 dB in our application (Symbol error rate $\approx 10^{-6}$).

The shaping-coder (scrambler) is applied in order to spread the effect of each shaping bit over many dimensions (signalling intervals). On the other hand the Tomlinson-Harashima precoder for channels with severe intersymbol interference (ISI) has memory enough to spread the influence of shaping bits, cf. [?]. Therefore, for ISI-channels shaping is possible without scrambling. But since the number of states of a Tomlinson-Harashima precoder with real coefficients is infinite, a sequential decoding procedure has to be applied instead of the Viterbi-algorithm. In fig. 5 shaping gains without scrambling are given for two shaping procedures. At first, a path search algorithm is used with an extension of the B best paths per step. For $B=4$ active states a net shaping gain of 0.8 dB, for $B=8$ of 0.95 dB can be achieved with shorter path registers than for shaping with scrambling. No loss due to descrambling has to be taken into account. If the more regular structure of a Viterbi-decoder is preferred, shaping without scrambling of data symbols is possible for ISI channels, too. The Viterbi-algorithm is adapted to an imaginary scrambler for sequence of shaping bits, whereas the mixture with the data symbols is leaved to the Tomlinson-Harashima-precoder. Using the same number of states, the gross gain is about 0.06 dB worse than for Trellis-Precoding. But this disadvantage is more than compensated by the fact, that no descrambling of data symbols is necessary at the receiver and, therefore, error propagation is avoided.

From (7) the distance gain $\Delta\ell_s$ by shaping is simply derived to:

$$\Delta\ell_s = G_s/A\left(\frac{\phi/f_0}{2IR}\right) \quad (12)$$

In Table 1, some factors $1/A(\phi/(2IRf_0))$ for the transformation of shaping gains into distance gains are listed.

6. CONCLUSIONS

Equation (7) offers a simple tool to estimate distance gains by multiple-duplex transmission, coding and shaping over single pair transmission for HDSL. Of course multiple-duplex transmission is the most powerful method to increase the maximum field length for HDSL. But in situations where the number of pairs is restricted, channel coding and shaping may help to avoid an intermediate signal regeneration. In combination these methods make possible to attach almost all subscribers via old twisted pair lines without signal regeneration.

7. REFERENCES

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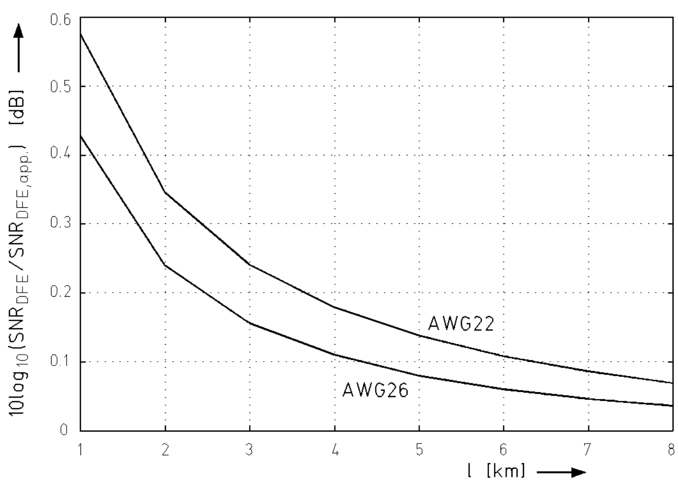

Fig. 1: Difference between SNR_{DFE} (eq. 4) and the approximation $SNR_{DFE,app.}$ (eq. 7) versus cable length

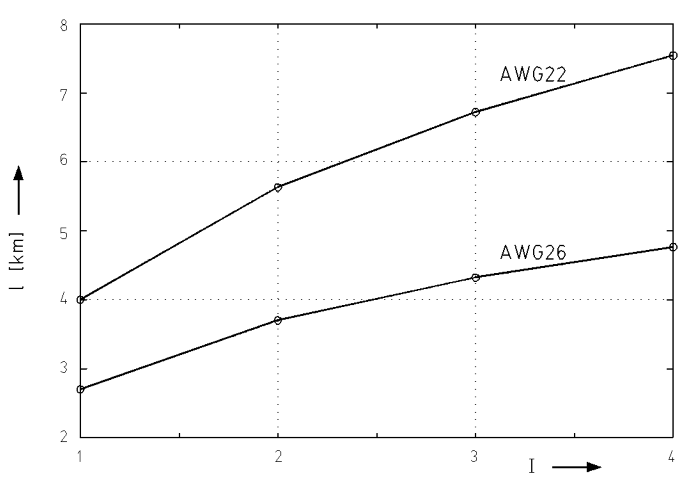

Fig. 2: Maximum distances for I-fold multiple-duplex transmission

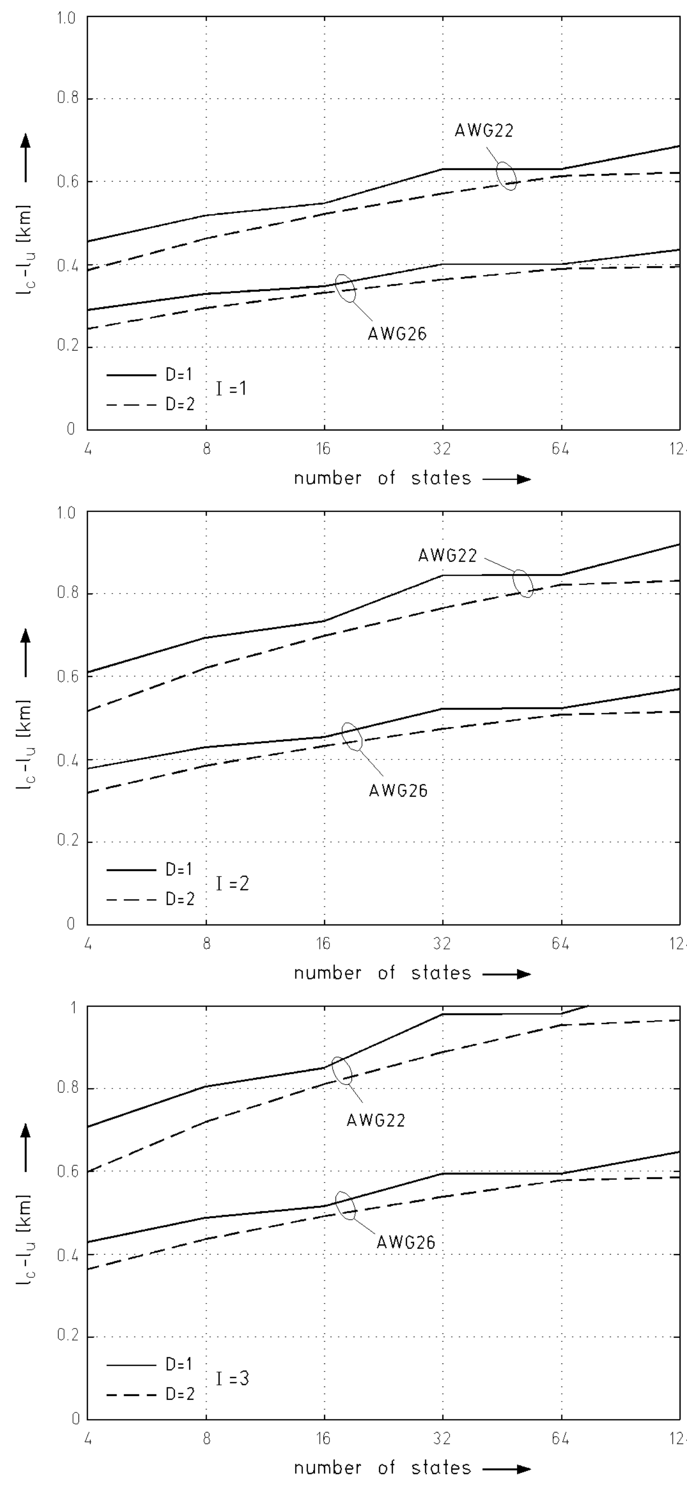

Fig. 3: Distance gains by one- (D=1) and two-dimensional (D=2) trellis-codes versus the number of encoder states

Table 1: Distance Gains [km] per dB net Shaping Gain

	0.4 mm (AWG 26)	0.6 mm (AWG 22)
Single Duplex ($I = 1$)	$0.087\frac{\text{km}}{\text{dB}}$	$0.137\frac{\text{km}}{\text{dB}}$
Double Duplex ($I = 2$)	$0.114\frac{\text{km}}{\text{dB}}$	$0.184\frac{\text{km}}{\text{dB}}$
Triple Duplex ($I = 3$)	$0.129\frac{\text{km}}{\text{dB}}$	$0.213\frac{\text{km}}{\text{dB}}$

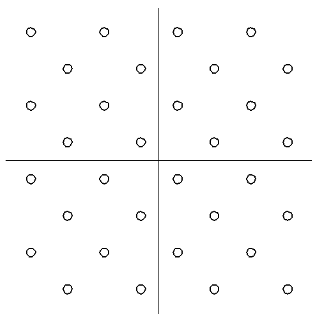

Fig. 4: Two-dimensional constellation with 32 points

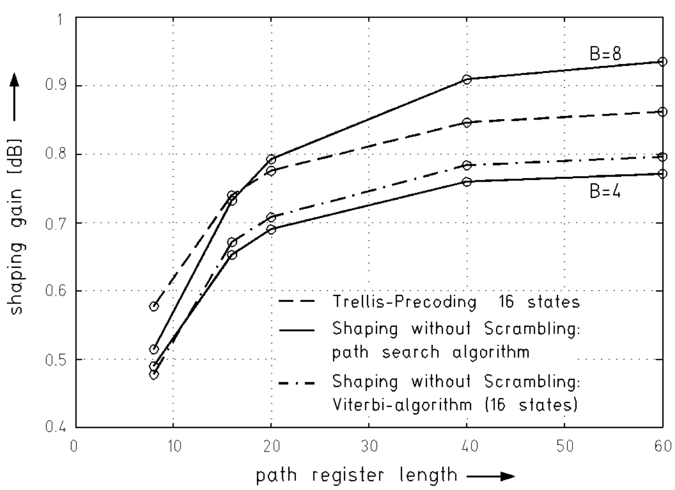

Fig. 5: Shaping gains (gross) versus path register length