Even-Integer Interference Precoding for Broadcast Channels

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Abstract

In this paper, an improved precoding strategy for broadcast channels, i.e., the transmission from one central transmitter (e.g., base station) to a number of distributed receivers (e.g., mobile terminals), is presented. The new scheme is a generalization of Tomlinson-Harashima precoding applied to multiple-input/multiple-output (MIMO) channels and a translation of concepts from lattice-reduction-aided detection in MIMO communication systems to the broadcast scenario. The main idea is not to suppress the multi-user interference completely, but to shape them to values which solely contribute to the anyway present periodic extension of the signal set. Although the method is generally applicable, we concentrate on situations with 2 users and binary signaling per real dimension. Here, the interferences are allowed to be taken from the even integers, hence we call the new scheme "even-integer interference precoding". Simulation results show that using the proposed technique, a considerable degree of diversity can be gained over linear preequalization, as well as conventional precoding.

1 Introduction

Over the last years it has been recognized that Tomlinson-Harashima precoding [16], [13], [3], originally proposed for the equalization of intersymbol-interference channels, is also well suited for suppression/cancellation of multi-user interference in multi-antenna as well as multi-user communication systems [2], [3], [9], [1]. This type of precoding at the transmitter can be viewed as the dual to decision-feedback equalization at the receiver; in the context of multi-user transmission known as successive cancellation. In broadcast scenarios (downlink situation), where no joint processing of the receive signals is possible, only this "MIMO precoding" is applicable.

Recently, an improved detection scheme for the multi-antenna setting was proposed by Yao and Wornell [18]. The main idea is the application of techniques known from lattice theory, in particular lattice (basis) reduction, i.e., the description of a lattice using a well suited basis. Using this scheme, significant gains over linear equalization and even the BLAST approach are possible. In particular, full diversity reception is achieved.

In this paper, we present an improved version of precoding for the broadcast scenario taking the ideas from lattice-reduction-aided equalization strategies into account. The main idea is not to suppress the multi-user interference completely, but shape them to values which solely contribute to the anyway present periodic extension of the signal set. Although the method is generally applicable, we concentrate on situations with 2 users and binary signaling per dimension. Here, the interferences are allowed to be taken from the even integers, hence we coin the term "even-integer interference precoding". The operation of this type of precoding is explained and simulation results are given to show the performance gains possibly by using the new method.

The organization of the paper is as follows: In Section 2 the channel model is given. Section 3 reviews briefly precoding for broadcast scenarios, and Section 4 discussed lattice-reduction-aided detection for multi-antenna systems. The new scheme is presented in detail in Section 5 and numerical results are given in Section 6. Conclusions are drawn in Section 7.

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2 Channel Model

We consider a simplified direct-sequence code division multiple access (DS-CDMA) downlink transmission scenario, where a base station, at which all K user signals are present, is communicating with K receivers distributed over some service area. Hence, in this broadcast scenario, communication takes place from a central transmitter (base station) to decentralized receivers (mobile terminals).

As usual, the end-to-end description of the above system (in complex baseband notation), is assumed to be modeled by a discrete-time flat-fading MIMO channel [1], [2], [3]

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{a} + \boldsymbol{n} \ . \tag{1}$$

Here $\boldsymbol{a}$ is the vector containing the complex-valued data symbols a_k, drawn from some signal constellation $\mathcal{A}$, of users $k = 1, 2, \ldots, K$, at a particular time instant. Since, due to the flat-fading assumption, each symbol interval can be processed by it own, we do not introduce any time index.

The complex transmission coefficients h_{kl}, collected in the $K \times K$ channel matrix $\boldsymbol{H}$, comprises spreading at the transmitter side, the underlying, possibly inter-chip-interference producing channel, individual matched filtering, and symbol-rate sampling at the receivers. Assuming the delay spread of the underlying channel to be short compared to the symbol duration, each symbol interval can still be processed separately. However, due to inter-chip-interference, the columns of $\boldsymbol{H}$ are non-orthogonal, hence equalization is required to detect the data symbols reliably.

The vector $\boldsymbol{n}$ denotes the effective (after individual matched filtering and sampling) additive white (complex) Gaussian noise. It is reasonable to assume that the noise samples are mutually uncorrelated and have the same variance, i.e.,[1] $\mathrm{E}\{\boldsymbol{n}\boldsymbol{n}^{\mathsf{H}}\} = \sigma_n^2 \boldsymbol{I}$.

In spite of the fact that the received symbols y_k are not available jointly, we collect them in the receive vector $\boldsymbol{y}$ and keep in mind that only operations describable by diagonal matrices are applicable. This is the main obstacle of this broadcast scenario compared to multi-antenna schemes.

3 Precoding for Decentralized Receivers

Meanwhile it is well-known that nonlinear precoding for the avoidance of multi-user interference at decentralized receivers can be derived from the dual problem, the separation of the users' signals at the base station in an uplink scenario, cf. [3], [5]. Here, successive cancellation is a popular strategy. This basic principle, also applied in the V-BLAST detection scheme [11], [6], can be interpreted as (zero-forcing) decision-feedback equalization (ZF-DFE) [10].

Roughly speaking, precoding can be derived from DFE by moving the feedback part to the transmitter. It has been shown (e.g., [2], [9], [1]) that Tomlinson-Harashima precoding [16], [13], originally proposed for the equalization of dispersive channels, is also well suited for the suppression/cancellation of multi-user interference. Since no joint processing of the receive signals is possible, all feedforward signal processing has to be moved to the transmitter, too. The resulting scheme is depicted in Figure 1.

The required matrices $\boldsymbol{F}$, $\boldsymbol{B}$, $\boldsymbol{P}$ and the scalar g may be obtained by a (sorted) QR-type decomposition [12] of the channel matrix $\boldsymbol{H}$, such that

$$\boldsymbol{P}^{\mathsf{T}} \boldsymbol{H} = \boldsymbol{B} \boldsymbol{F}^{-1} g^{-1} \; . \tag{2}$$

[1]Notation: $\boldsymbol{A}^{\mathsf{T}}$ is the transpose of matrix $\boldsymbol{A}$, $\boldsymbol{A}^{\mathsf{H}}$ the Hermitian (i.e., conjugate) transpose. $\boldsymbol{I}$: identity matrix. $\mathrm{E}\{\cdot\}$: expectation.

Here, $\boldsymbol{B} = [b_{kl}]$ is a unit-diagonal lower triangular matrix, $\boldsymbol{F}$ is a matrix with orthogonal columns, and $\boldsymbol{P}$ is a permutation matrix (a single 1 in each row and column which gives a sorting of the rows of $\boldsymbol{H}$), equivalent to the processing order. The optimum sorting and the matrices $\boldsymbol{B}$ and $\boldsymbol{F}$ may be obtained from the V-BLAST detection algorithm [11], [6] applied to $\boldsymbol{H}^{\mathsf{H}}$.

The above factorization is done under two assumptions: First, all users should perform the same. Hence, receiver-side scaling (automatic gain control for unit-gain signal transmission) is given by a single scalar g, identical for all users. Implementing precoding in this way, each user experiences the same additive white Gaussian noise (AWGN) channel with signal-to-noise ratio (SNR) $\sigma_a^2/(g^2\sigma_n^2)$ at the input of the slicer. Second, in order to fix total transmit power to the same level as the direct transmission of the data vector $\boldsymbol{a}$, $\boldsymbol{F}$ is normalized such that $\mathrm{trace}\{\boldsymbol{F}\boldsymbol{F}^{\mathsf{H}}\} = K$. Note, here we fix the average power for each channel realization to the given value (short-term constraint) and do not relax the constraint such that only a fixed transmit power on average over all channel realizations is guaranteed (long-term constraint).

If a 4-QAM constellation for the data symbols a_k is used, i.e., $a_k \in \{\pm\frac{1}{2} \pm \frac{1}{2}\mathrm{j}\}$, the operation of the precoder is as follows (the generalization to other signal sets is straightforward, see [3]): Given the data symbols a_k, the symbols $\tilde{x}_k$ are calculated successively as

$$\tilde{x}_1 \;=\; a_1 \tag{3a}$$

$$\tilde{x}_k \;=\; a_k + d_k - \sum_{l=1}^{k-1} b_{kl}\tilde{x}_l \; , \quad k = 2, \ldots, K \; . \tag{3b}$$

Here, the so-called precoding symbols d_k taken from the even integer grid ($d_k \in 2\mathbb{Z}^2$) are chosen implicitly by the modulo device such that the real and imaginary part of the resulting symbol $\tilde{x}_k$ fall into the (half-open) interval $[-1, 1)$. The operation of the precoder can hence be interpreted as feeding effective data symbols $v_k \overset{\mathrm{def}}{=} a_k + d_k$ into the linear predistortion filter $\boldsymbol{F}\boldsymbol{B}^{-1}$, which is present if the modulo device in the feedback loop is ignored [3]. In other words, the initial signal constellation is extended periodically to the entire complex plane. All signal points, whose real and imaginary parts are spaced by integer multiples of 2 (even integers) are congruent. From these equivalent points, the unique point which results in a predistorted symbol with minimum possible magnitude of real and imaginary part is selected.

Precoding increases average transmit power only slightly [15], [3]. Moreover, because of the above given normalization, average power is not changed by the matrix $\boldsymbol{F}$. Using 4-QAM modulation, the highest increase in average transmit power—which here calculates to $\mathrm{E}\{\boldsymbol{x}^{\mathsf{H}}\boldsymbol{x}\} \approx 2K/3$ compared to $\mathrm{E}\{\boldsymbol{a}^{\mathsf{H}}\boldsymbol{a}\} \approx 2K/4$—occurs.

At the receiver side, the effective data symbols v_k corrupted by additive noise with variance $g^2\sigma_n^2$ but free

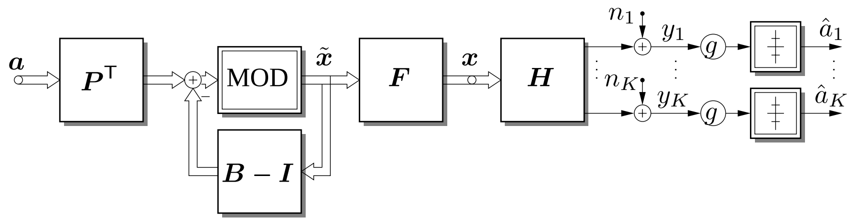

Fig. 1. Precoding for decentralized receivers. (The decision devices take the modulo congruence into account.)

of multi-user interference are visible at the input of the decision device. Using a slicer, which takes the periodic extension into account, estimates $\hat{a}_k$ on the data symbols can be generated.

The above discussion reveals that performance of this interference-avoidance scheme is governed by the factor g in the factorization (2) of the channel matrix $\boldsymbol{H}$. For random channel matrices with independent zero-mean unit-variance complex Gaussian coefficients h_{kl}, g will be Rayleigh distributed, which, in turn, results in the same performance of the users as for a (single-input/single-output) interference-free flat Rayleigh fading channel.

4 Lattice-Reduction-Aided Detection and Precoding

Recently, Yao and Wornell [18], [19] proposed a scheme for improved detection in multi-antenna systems (joint processing at the receiver side is possible), which we now briefly review. The idea is to combine principles known from lattice theory, in particular *lattice (basis) reduction*, i.e., the choice of a more suited representation of a lattice, with conventional low-complexity detectors. Originally, simple linear equalization has been proposed, but additional gains are possible by using decision-feedback equalization or precoding [17]. Surprisingly, the error rate curves when using such detectors run parallel to those for maximum-likelihood (ML) detection. After having performed the costly preprocessing step—finding the reduced basis, e.g., using the LLL algorithm [14], [7]—detection complexity is very modest.

In order to apply the techniques known from lattice theory, the complex-valued MIMO channel model (1) is equivalently written as a $2K$-dimensional real-valued MIMO channel model according to

$$\begin{bmatrix} \mathrm{Re}\,\boldsymbol{y} \\ \mathrm{Im}\,\boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\,\boldsymbol{H} & -\mathrm{Im}\,\boldsymbol{H} \\ \mathrm{Im}\,\boldsymbol{H} & \mathrm{Re}\,\boldsymbol{H} \end{bmatrix} \begin{bmatrix} \mathrm{Re}\,\boldsymbol{x} \\ \mathrm{Im}\,\boldsymbol{x} \end{bmatrix} + \begin{bmatrix} \mathrm{Re}\,\boldsymbol{n} \\ \mathrm{Im}\,\boldsymbol{n} \end{bmatrix}, \tag{4}$$

($\mathrm{Re}\,\cdot$ and $\mathrm{Im}\,\cdot$ denote real and imaginary part), or in short using obvious definitions

$$\boldsymbol{y}_{\mathrm{r}} = \boldsymbol{H}_{\mathrm{r}}\boldsymbol{x}_{\mathrm{r}} + \boldsymbol{n}_{\mathrm{r}} \; . \tag{5}$$

In this paper, we will use real- and complex-valued channel model interchangeably; the respective context determines whether complex operations in the equivalent baseband domain or real-valued operations (which is more general since real and imaginary part can be treated differently) are meant.

Since in digital communications it is common to drawn the signal points from (a translate of) the integer lattice $\mathbb{Z}^2$, each quadrature component is taken from the integer lattice $\mathbb{Z}$. In turn, the noise-less receive points are given by (a subset of) the lattice $\boldsymbol{H}_{\mathrm{r}}\mathbb{Z}^{2K}$. Detection is hence the task of finding the lattice point closest to the (noisy) receive vector $\boldsymbol{y}_{\mathrm{r}}$. Using low-complexity detection schemes, this, however, can be done more reliably (less noise enhancement) if the columns of $\boldsymbol{H}_{\mathrm{r}}$ are close to orthogonal.

Applying lattice (basis) reduction [14], [7] which factors the channel matrix $\boldsymbol{H}_{\mathrm{r}}$ as

$$\boldsymbol{H}_{\mathrm{r}} = \boldsymbol{H}_{\mathrm{red}}\,\boldsymbol{R} \; , \tag{6}$$

where $\boldsymbol{R}$ is a matrix with integer entries that has unit determinant, i.e., $\boldsymbol{R}^{-1}$ also contains only integer entries, results in a more suited channel description $\boldsymbol{H}_{\mathrm{red}}$. This matrix is a "reduced" basis for the same lattice, i.e., $\boldsymbol{H}_{\mathrm{r}}\mathbb{Z}^{2K} \equiv \boldsymbol{H}_{\mathrm{red}}\mathbb{Z}^{2K}$, but its columns are close (or at least closer) to orthogonal.

Instead of linear equalization of the entire channel $\boldsymbol{H}_{\mathrm{r}}$, now only the factor $\boldsymbol{H}_{\mathrm{red}}$ is linearly equalized. Since $\boldsymbol{R}\mathbb{Z}^{2K} = \mathbb{Z}^{2K}$ the (noise-free) decision symbols are drawn from the integer grid, and individual threshold decision in each component to the integer grid can be performed. Since the noise enhancement due to $\boldsymbol{H}_{\mathrm{red}}^{-1}$ is lower than that of $\boldsymbol{H}_{\mathrm{r}}^{-1}$, a performance gain is achieved. Finally, to recover data, via $\boldsymbol{R}^{-1}$ estimates $\hat{a}_k$ of the initial data symbols are generated. In summary, linear equalization followed by threshold decision is replaced by partial linear equalization, threshold decision, followed by residual equalization, which, fortunately, can be done without noise enhancement.

The transmission scheme is depicted in the upper part of Figure 2. Instead of performing linear equalization at the receiver side, lattice-reduction-aided equalization can also be done via DFE, as linear preequalization, or via precoding at the transmitter side. Thereby, all equalization schemes operate only on the reduced channel matrix $\boldsymbol{H}_{\mathrm{red}}$. Noteworthy, for transmitter-side techniques, decomposition (6) has to be replaced by the transposed form [17]

$$\boldsymbol{H}_{\mathrm{r}} = \boldsymbol{R}\,\boldsymbol{H}_{\mathrm{red}} \; , \tag{7}$$

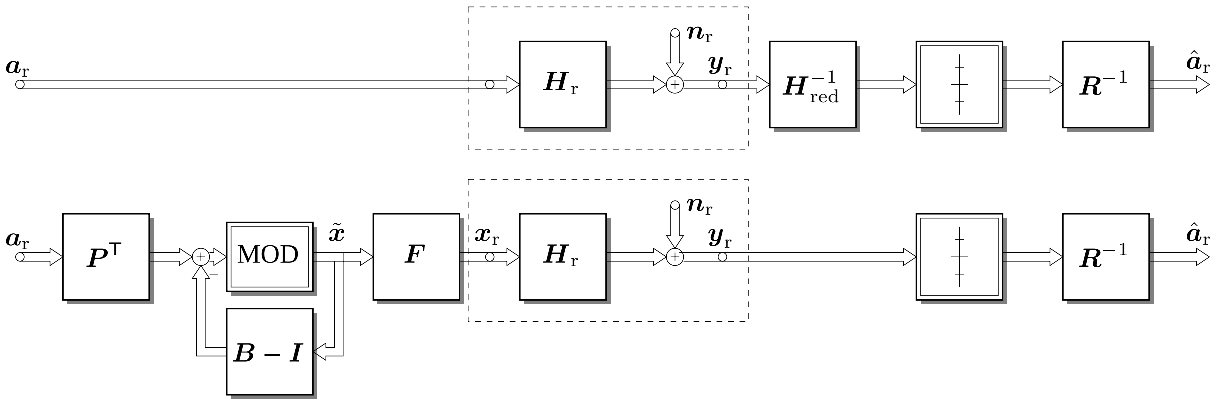

Fig. 2. Lattice-reduction-aided equalization of MIMO channels. Top: linear equalization at the receiver side. Bottom: precoding at the transmitter side. (The decision devices work on the entire (shifted) integer grid.)

i.e., lattice reduction is performed on $\boldsymbol{H}_\mathrm{r}^\mathsf{T}$. Using precoding, the "reduced" channel matrix is further decomposed, such that (cf. (2))

$$\boldsymbol{P}^\mathsf{T}\boldsymbol{H}_\mathrm{red} = \boldsymbol{B}\boldsymbol{F}^{-1}g^{-1} \;, \qquad (8)$$

where the matrices $\boldsymbol{B}$, $\boldsymbol{F}$, $\boldsymbol{P}$ and the scalar g have the properties discussed above.

However, regardless of the actual equalization strategy (linear/nonlinear; receiver/transmitter side), joint processing at the receiver side (matrix $\boldsymbol{R}^{-1}$ succeeding the slicer) is required. Unfortunately, this is irreconcilable with broadcast scenarios and lattice-reduction-aided equalization is limited to the multi-antenna case.

5 Even-Integer-Interference Precoding

In order to apply the basic idea of lattice-reduction-aided equalization for broadcast channels (transmission over MIMO channels with distributed receivers), we observe the following facts:

(i) Using precoding with binary signaling in each real component, all points spaced by integer multiples of 2 at the receiver side represent the same binary symbol.

(ii) Since $\boldsymbol{H}_\mathrm{red}$ is equalized, the remaining interference is given by the matrix $\boldsymbol{R}$ in the factorization (7) of the channel matrix $\boldsymbol{H}_\mathrm{r}$.

If the matrix $\boldsymbol{R}$ would have unit diagonal and all other elements would be even integers, i.e., $\boldsymbol{R} = [r_{kl}]$, with $r_{kk} = 1$, and $r_{kl} \in 2\mathbb{Z}$, $k \neq l$, the residual interferences (ignoring for the moment a constant shift) would also be multiples of 2. However, this does not contribute to the disturbance as it can be accounted to the periodic extension and is anyway eliminated by the modulo reduction inherent in the decision process. That is, instead of completely avoiding or suppressing interference, it is driven to a well suited multiple of 2. Since, for binary signaling, the interferences are allowed to be taken from the even integers, we term this transmission scheme "*even-integer interference precoding*".

In summary, using binary transmission per real dimension, precoding for broadcast scenarios can be done as follows. First, the (real-valued, equivalent) channel matrix is factorized as[2]

$$\boldsymbol{H}_\mathrm{r} = \boldsymbol{R}\,\boldsymbol{H}_\mathrm{red} \quad \text{with} \quad \boldsymbol{R} = \begin{bmatrix} 1 & & 2\mathbb{Z} \\ & \ddots & \\ 2\mathbb{Z} & & 1 \end{bmatrix} . \qquad (9)$$

(Contrary to conventional lattice reduction (6), here $\boldsymbol{R}$ is not required to have unit determinant, but from a capacity point of view, a unit-determinant matrix $\boldsymbol{R}$ is desirable.) The "reduced" channel matrix is further decomposed as

$$\boldsymbol{P}^\mathsf{T}\boldsymbol{H}_\mathrm{red} = \boldsymbol{B}\boldsymbol{F}^{-1}g^{-1} \;, \qquad (10)$$

where $\boldsymbol{B}$ is a unit-diagonal lower triangular matrix, $\boldsymbol{F}$ is a matrix with orthogonal columns, and $\boldsymbol{P}$ is a permutation matrix.

Figure 3 (top) shows the transmission scheme using even-integer-interference precoding; the other figures show the linear description and the resulting transmission scheme. The blocks "$\mathbb{C}/\mathbb{R}$" and "$\mathbb{R}/\mathbb{C}$" denote conversion from complex-valued to real-valued description and conversion from real-valued to complex-valued description (see (4) and (5)), respectively.

Since precoding (neglecting the periodic extension) equalizes $\boldsymbol{H}_\mathrm{red}$, only $\boldsymbol{R}\boldsymbol{P}/g$ remains. The gain factor is compensated at the receiver side (automatic gain control at each receiver) and the permutation by $\boldsymbol{P}$ can be eliminated by permuting the components of the data vector $\boldsymbol{a}_\mathrm{r}$ by $\boldsymbol{P}^\mathsf{T}$, i.e., feeding $\tilde{\boldsymbol{a}}_\mathrm{r} = \boldsymbol{P}^\mathsf{T}\boldsymbol{a}_\mathrm{r}$ into the precoder. We remind, that the entire processing at the precoder is based on real-valued vectors of dimension $2K$. Real and imaginary parts of the data symbols are separated and processed individually,

[2] The design of an efficient algorithm which performs this required decomposition is topic of further research.

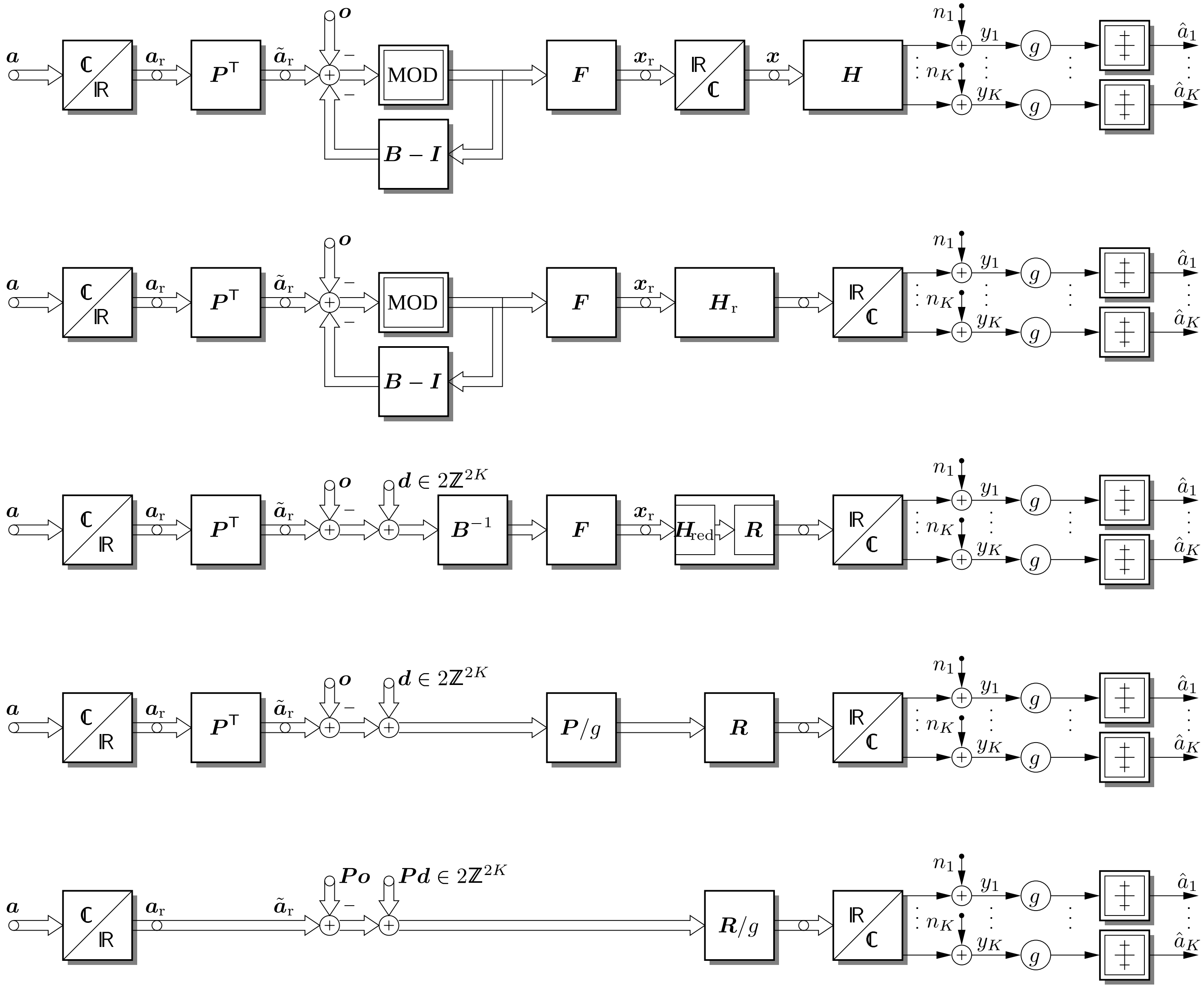

Fig. 3. Even-integer-interference precoding for broadcast channels (top) and its linearized interpretation using equations (9), (10), and (11). (The decisions devices take the modulo congruence into account.)

giving more degrees of freedom than the joint complex-valued processing, a fact also used in so-called *widely-linear equalization* schemes [8].

Finally, because the signal points in each dimension are not chosen from the integer lattice but from a translate of the lattice $\mathbb{Z}$ (translation by $1/2$), at the receiver a systematic offset

$$\boldsymbol{R}[1/2 \cdots 1/2]^{\mathsf{T}}$$

would be present. This offset can be compensated already prior to the precoder by subtracting

$$\boldsymbol{o} = \boldsymbol{P}^{\mathsf{T}}(\boldsymbol{I} - \boldsymbol{R}^{-1})[1/2 \cdots 1/2]^{\mathsf{T}} . \qquad (11)$$

Since the modulo device restricts the components of $\tilde{\boldsymbol{x}}$ always to the prescribed range, this offset compensation can be done without any additional increase in average transmit power.

The principle presented above for binary signaling can straightforwardly be extended to general QAM transmission. Assuming a square QAM constellation with a number of M_k^2 signal points (with spacing 1) for user k, only the $2K \times 2K$ matrix $\boldsymbol{R}$ in (9) has to be replaced. Now, the off-diagonal elements in row k have to be integer multiples of M_k. Keeping (4) in mind, we now require

$$\begin{aligned} \boldsymbol{R} = [r_{kl}] \;, &\qquad \text{with } r_{kk} = 1, \quad \text{and} \qquad (12)\\ r_{kl} \in M_k\mathbb{Z}, &\qquad k = 1, \ldots, K, \quad k \neq l, \\ r_{k+K,l} \in M_k\mathbb{Z}, &\qquad k = 1, \ldots, K, \quad k + K \neq l \;. \end{aligned}$$

6 Numerical Results

The performance gains possible by the proposed precoding schemes over other equalization schemes are now shown by numerical simulations. We restrict ourselves to $K = 2$ users, a situation which arises frequently in practical systems where two dominant (high-rate) users are present beside a number of other, low-rate users, contributing to the additive channel noise. For both users 4-QAM signaling is chosen and the channel matrix is randomly selected. The elements of the complex-valued channel model $\boldsymbol{H}$ are unit-variance complex Gaussian distributed and constant over one transmission burst. The results are obtained by averaging over a sufficiently large number of different channel realizations. Since up to now no efficient algorithm is known to perform the factorization (9), this step is done by applying Monte-Carlo methods. Among a number of tested factorizations, the one resulting in the smallest gain factor g is selected.

In Figure 4 the average bit error rate (BER) of the users is depicted over the average transmitted energy per bit $\bar{E}_\mathrm{b}$ divided by the (one-sided) noise power spectral density N_0. Note, due to the specific factorization (2) both users exhibit the same performance.

It is visible that all (nonlinear) precoding schemes outperform linear preequalization. Noteworthy, the real-valued processing in precoding schemes—based on the matrix $\boldsymbol{H}_\mathrm{r}$—achieves a better performance than the obvious complex-valued processing based on $\boldsymbol{H}$. This is due to the fact, that a separation of real and imaginary part of the symbols allows for a different ordering of the components (permutation matrix $\boldsymbol{P}$) and leads to a somewhat smaller gain factor g, for details see [4].

Even-integer-interference precoding based on decomposition (9) of the (real-valued) channel matrix provides further gains. For reference only, the bit error rate for lattice-reduction-aided linear equalization which requires joint processing at the receiver, is shown, too. A comparison of the curves shows that using the proposed precoding technique a significant degree of diversity is gained. The error rate curves of linear preequalization and that of standard precoding show the typical behavior of flat Rayleigh fading channels, i.e., the slope is one decade in error rate per 10 dB in SNR. Lattice-reduction-aided linear equalization according to [18] exhibits the full diversity of 2. The new scheme has a diversity order in the range of 1.75. Especially at low error rates significant gains over conventional techniques are possible.

In order to illustrate the potential for gains, we consider the following $K = 2$ complex-valued MIMO system, with channel matrix

$$\boldsymbol{H} = \begin{bmatrix} 0.6951 + 0.1736\mathrm{j} & 0.2795 + 0.0072\mathrm{j} \\ 0.9471 + 0.3104\mathrm{j} & 0.3918 + 0.0794\mathrm{j} \end{bmatrix} . \qquad (13)$$

QR-type decomposition according to (2) for precoding based on complex-valued processing results in

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (14a)$$

$$\boldsymbol{F} = \begin{bmatrix} 0.0645 - 0.0161\mathrm{j} & 0.0541 + 0.5106\mathrm{j} \\ 0.0259 - 0.0007\mathrm{j} & 0.1491 - 1.3073\mathrm{j} \end{bmatrix} \qquad (14b)$$

$$\boldsymbol{B} = \begin{bmatrix} 1.0000 & 0.0000 \\ 1.3902 + 0.1196\mathrm{j} & 1.0000 \end{bmatrix} \qquad (14c)$$

$$g = 18.21 \,. \qquad (14d)$$

If precoding is done by separating real and imaginary part, the same factorization applied to the corresponding real 4×4 matrix $\boldsymbol{H}_\mathrm{r}$ gives

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (15a)$$

$$\boldsymbol{F} = \begin{bmatrix} 0.0844 & -0.1389 & 0.0296 & -0.6682 \\ 0.0340 & 0.0234 & 0.0242 & 1.7107 \\ -0.0211 & -0.4977 & 0.1644 & 0.0708 \\ -0.0009 & -0.5000 & -0.1707 & 0.1951 \end{bmatrix} \qquad (15b)$$

$$\boldsymbol{B} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 1.3902 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & -7.0937 & 1.0000 & 0.0000 \\ 0.1196 & -9.8617 & 1.3902 & 1.0000 \end{bmatrix} \qquad (15c)$$

$$g = 13.92 \,. \qquad (15d)$$

Hence, comparing complex-valued and real-valued processing, an asymptotic gain of

$$20 \cdot \log_{10}(18.21/13.92) = 2.33 \text{ dB}$$

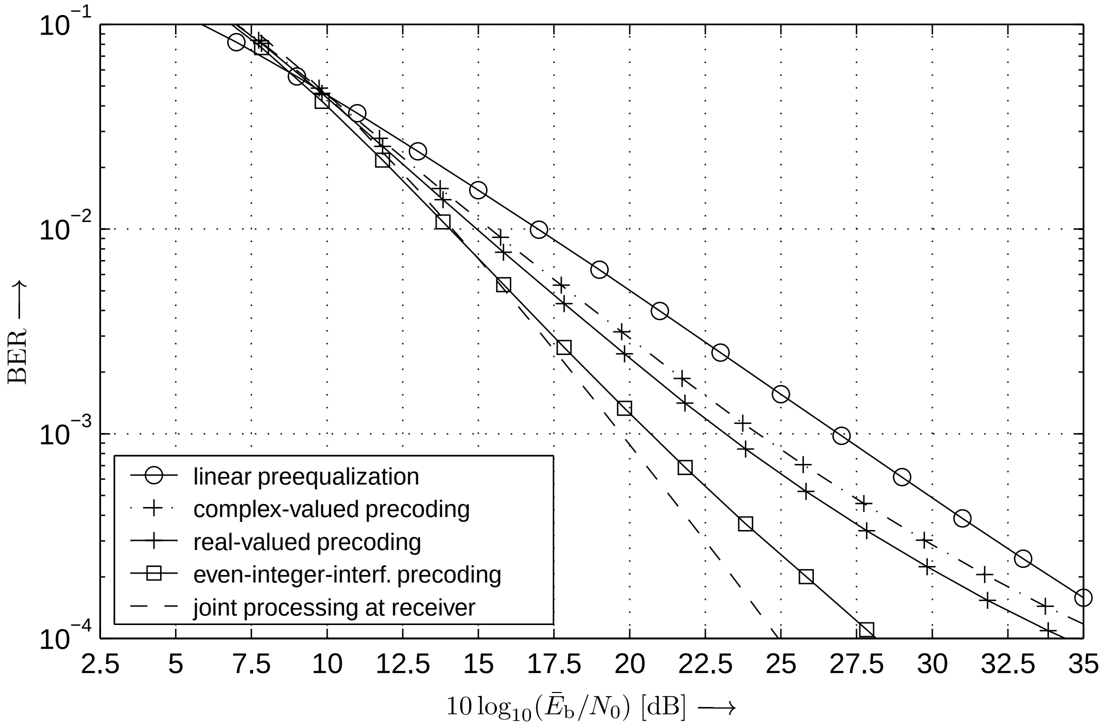

Fig. 4. Bit error rate over the average transmitted energy per bit $\bar{E}_\mathrm{b}$ divided by the (one-sided) noise power spectral density N_0 for (top to bottom) linear preequalization, complex-valued precoding, real-valued precoding, and even-integer-interference precoding. Reference: lattice-reduction-aided equalization using joint processing at the receiver. $K = 2$; binary transmission per real component.

is achieved. In this situation the processing order is changed from "user 1 (real and imaginary part) – user 2 (real and imaginary part)" as done in complex-valued processing to (cf. (4)) "user 1 real part – user 2 real part – user 1 imaginary part – user 2 imaginary part".

Factoring the channel matrix $\boldsymbol{H}_\mathrm{r}$ according to (9) as

$$\begin{bmatrix} 0.6951 & 0.2795 & -0.1736 & -0.0072 \\ 0.9471 & 0.3918 & -0.3104 & -0.0794 \\ 0.1736 & 0.0072 & 0.6951 & 0.2795 \\ 0.3104 & 0.0794 & 0.9471 & 0.3918 \end{bmatrix} = \qquad (16)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.6951 & 0.2795 & -0.1736 & -0.0072 \\ -0.4431 & -0.1672 & 0.0368 & -0.0650 \\ -1.2166 & -0.5518 & 1.0423 & 0.2939 \\ 2.7436 & 1.1830 & -1.1376 & -0.1959 \end{bmatrix}$$

and performing the QR-type decomposition for the last matrix ($\boldsymbol{H}_\mathrm{red}$) gives

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (17a)$$

$$\boldsymbol{F} = \begin{bmatrix} -0.2124 & -0.0004 & -0.1156 & -0.5797 \\ -0.0801 & 0.0739 & 0.0556 & 1.6375 \\ 0.0176 & -0.4979 & -0.3908 & 0.3183 \\ -0.0312 & -0.4691 & 0.4237 & -0.0793 \end{bmatrix} \qquad (17b)$$

$$\boldsymbol{B} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ -1.5688 & 1.0000 & 0.0000 & 0.0000 \\ -6.2750 & 1.2426 & 1.0000 & 0.0000 \\ 3.4452 & 2.3257 & -1.5688 & 1.0000 \end{bmatrix} \qquad (17c)$$

$$g = 9.07\,. \qquad (17d)$$

Hence a gain of

$$20 \cdot \log_{10}(13.92/9.07) = 3.72 \text{ dB}$$

compared to precoding employing real-valued processing and even a gain of

$$20 \cdot \log_{10}(18.21/9.07) = 6.05 \text{ dB}$$

compared to (conventional) complex-valued processing is possible.

7 Conclusions

In this paper, improved precoding for broadcast channels, i.e., a central transmitter is communicating with decentralized receivers, has been presented. Key idea is not to suppress the interferences completely, but to shape them to values which contribute to the periodic extension of the signal set, anyway present in precoding schemes. In this paper we have concentrated the discussion to situations with 2 users and binary signaling per dimension. Since here the interferences are allowed to be even integers, hence we call the transmission scheme "even-integer interference precoding". It has been shown by simulations that substantial performance gains over linear preequalization are achievable. In particular, even if no joint processing at the receiver side is possible, the newly proposed strategy can gain in diversity.

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